Challenge Standards for Student Success

Mathematics

Contents

Introduction

The model mathematics standards presented in this publication identify the content that students need to learn and illustrate the type of work they need to do at each grade level to show that they are developing proficiency in mathematics.

Standards are not curriculum. They do not describe a full and comprehensive mathematics program or prescribe the kinds of experiences students will need to develop a good understanding of mathematics. Students need sufficient time, usually across several grade levels, to develop the proficiency required to meet a standard that is included at a particular grade.

These standards build on those developed by the New Standards[1] project and those developed by the National Council of Teachers of Mathematics (NCTM), which appear in *Curriculum and Evaluation Standards for School Mathematics* (1989). The standards are also closely aligned with the *Mathematics Framework for California Public Schools* (California Department of Education, 1992). Six standards have been identified for every grade level. They cover the same content areas as do six of the eight standards used by the New Standards project.[2] The six standards are as follows:

1. Number and Operations
2. Geometry and Measurement
3. Function and Algebra
4. Statistics and Probability
5. Problem Solving and Mathematical Reasoning
6. Mathematical Communication

---

[1] New Standards is a collaboration of the Learning Research and Development Center of the University of Pittsburgh and the National Center on Education and the Economy; in partnership with states and urban districts, it is working to build an assessment system to measure students' progress toward meeting national standards at levels that are internationally benchmarked.

[2] The New Standards' other two standards, "Mathematical Skills and Tools" and "Putting Mathematics to Work," have been incorporated into the six standards presented in this document.

Each standard at every grade level, kindergarten through high school, is composed of two parts:

Statement of the standard. The general statement for each standard is the same across all grade levels, kindergarten through high school. The bulleted items following the statement for each standard identify examples of what students at the particular grade level should know and be able to do to meet the standard. In the first four standards, work examples are differentiated by grade level and are cumulative—the assumption is that students can do work that meets the standards for earlier grades. In two standards, those of problem solving and mathematical reasoning and of mathematical communication, the work examples are similar across grade spans, kindergarten through grade four and grades five through eight, and for each set of high school standards.

Sample assignments or tasks related to the standard. Several samples of assignments or tasks follow the statement of each standard. These assignments or tasks are representative of the kind of work that students at the specified grade level should be able to complete satisfactorily. The samples are not necessarily intended to be assessment items, nor are they as a whole intended to be a comprehensive set of activities or tasks that cover the full domain of the content area.

High School Standards

The mathematics standards for high school identify the important mathematics that students need to know and be able to do by the time they graduate. The standards are based on the assumption that students have completed at least two years of rigorous mathematics courses that will prepare them for citizenship, for the workplace, and for further study.

Many schools continue to offer algebra and geometry as the first two courses in the college-preparatory sequence. Other schools, building on the recommendations of the *Mathematics Framework for California Public Schools* and the NCTM's *Curriculum and Evaluation Standards for School Mathematics,* now offer a sequence of core courses in which the mathematical content of each course is more integrated. To meet the needs of all schools in California, this document presents the mathematics standards for high school in three sets: (1) algebra; (2) geometry; and (3) an integrated sequence. Students will need to meet the standards for both algebra *and* geometry *or* the standards for the integrated sequence. The standards are the same whether students take algebra *and* geometry *or* the integrated sequence; the standards for the integrated sequence are identical to the combined standards for algebra and geometry.

Samples of Student Work

A few samples of student work, along with commentaries describing the work and linking it with the standards, are provided after the standards for grades four, eight, and high school. (The intent is to include samples of student work at all grade levels in future editions of this document.) The collection of work is illustrative of high-quality work that can be done by students at a particular grade level. However, the work samples do not establish levels of performance. A greater quantity and variety of work would be necessary to determine whether a student meets the standards.

Kindergarten

Standard 1. Number and Operations

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in number and operations.

For example, students in kindergarten who meet the standard will:

- Count by rote up to 30.
- Estimate the number of and count up to 20 objects.
- Match the numeral with a collection of up to ten objects.
- Compare two groups of objects (up to ten objects in each group) and identify which group has more in it.
- Combine groups of objects and count the total up to ten.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Count how many people brought something from home to share with the class today.
2. Are there enough pencils in this box so that everyone at the table can have one?
3. Vanessa brought three cans for the food drive, and Emilio brought two cans. How many cans of food is that?

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in kindergarten who meet the standard will:

- Sort and classify simple two-dimensional shapes.
- Compare two objects by length and identify which is longer or shorter or determine whether they are about the same.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Who is taller, Kim or Tyrone?
2. Match the lids with the containers. Which one goes on the margarine tub? Which goes on the shoe box?
3. Which blocks can you stack to make a tower? Which blocks roll? Which blocks can be put together with no spaces between?

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in kindergarten who meet the standard will:

- Identify, extend, and build simple patterns.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. What comes next in these patterns?
   Using beads. Red, red, blue, red, red, blue . . .
   Moving and saying the motion aloud. Jump, clap, jump, stretch, jump, clap . . .
   Arranging sticks. | | \ \ | | \ \ . . .
2. The yellow and green tiles make a checkerboard pattern. Use more tiles to make the pattern bigger.
3. Count the people in this group but say only every other number out loud. For example, as you point to each person, say “one” to yourself, say “two” out loud, say “three” to yourself, and so on.

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in kindergarten who meet the standard will:

- Participate in the construction of concrete and pictorial graphs.
- Pose questions for collecting data based on the immediate environment.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. In what ways do shoes fasten? Is anyone wearing shoes that do not have a fastener? Do more students in the room wear shoes that tie or shoes that fasten with Velcro? Let the children with these kinds of shoes line up so that the class can see.
2. What kind of pets do you have? Do more children have cats or do more have dogs?

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in kindergarten who meet the standard will:

- Use concepts and skills acquired through previous activities.
- Explain what materials to use, when possible.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in kindergarten who meet the standard will:

- Find a solution that is correct and makes sense.
- Tell what their solution is and how they know it works.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in kindergarten who meet the standard will:

- Show the solution with another material or in another way when directed to do so. For example, show how a strategy for sharing a given number of stickers can be used again when sharing beads for stringing necklaces.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Make a pattern by using some of these materials: pattern blocks, buttons, color tiles, connecting cubes, beads, and so forth.
2. Here are some stickers for your group. Share them so that everyone has the same amount.
3. Make a greeting card. What materials will you need, and how much of each material will you need?

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of mathematical communications of others.

For example, students in kindergarten who meet the standard will:

- Use appropriate mathematical vocabulary; for example, the words for simple shapes, attributes, and numbers.
- Show ideas by using a variety of concrete materials, such as connecting cubes, pattern blocks, buttons, beads, and color tiles, and by pasting paper representations of materials.
- Explain strategies used in solving problems.
- Share ideas when probed by the teacher or dictate your ideas or strategies to an adult for recording.
- Understand and follow oral directions for appropriate mathematical activities.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

Use oral language and physical materials to show how to take numbers apart in order to solve problems when using mental mathematics. For example, a student might say, “I have two red buttons and three yellow buttons, and that makes five” or “I have four blocks; so I need one more to have five.”

Grade One

Standard 1. Number and Operations

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in number and operations.

For example, students in grade one who meet the standard will:

- Compare two groups of objects (up to 20 objects in each group) and identify which group has more or less in it and count how many more are in one group.
- Combine groups of objects up to a total of 20 objects and write the correct addition equation.
- Count orally by ones and tens to 100 and by twos and fives to 50 and write numerals to 50.
- Separate up to 20 objects into two groups and write the correct subtraction equation.
- Figure out all addition facts (sums to ten) and related subtraction facts, with and without the use of concrete objects, orally and in writing.
- Identify halves and wholes by using concrete objects in simple situations.
- Identify and give the value of common coins.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. How many more girls than boys are there in the class?
2. Count the buttons. How many are there? Now put some of them in one pile and the rest in another pile. How many objects are in each pile? How many altogether?
3. There were 12 crayons in the box, now there are eight. How many are missing?
4. Hand me half of the cookie.
5. Here are eight marbles. Give Mario half and you keep half. How many do you each get?

Standard 2. Geometry and Measurement

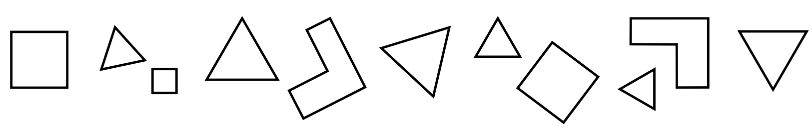
Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade one who meet the standard will:

- Sort and classify two- and three-dimensional shapes.
- Use words, such as “in front of” and “behind,” “right” and “left,” and “above” and “below,” to represent the position and direction of objects.
- Use nonstandard units to measure length by using a number of the units end to end.
- Order three objects by length.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Reach inside the bag and, without looking, describe something about the object in the bag. Is it round, flat, square, a cube, a ball, a can? Reach inside the other bag and see whether you can find an identical object.
2. Are the following shapes alike or different? How are they alike? How are they different? Are there any shapes that look the same but are different in size? Do any of them look as though they will fit on top of each other? What if you turned them around or turned them over? Cut them out and see whether you can make them fit exactly on one another.



3. How many paper clips do you need to measure the length of the book?

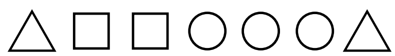
Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in grade one who meet the standard will:
- Identify, extend, and build patterns.
- Translate patterns from one material or symbol to another.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Here is a pattern using shapes:

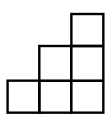
   Can you change it to a pattern using colors?

   Example: Red, blue, blue, yellow, yellow, yellow, red

   How can you do the same pattern in a different way?

   Example: A B B C C C A

2. Build a staircase with blocks as shown below. The first step has one block, the second step has two blocks, and so forth. How many blocks will there be in the seventh step?

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade one who meet the standard will:

- Use concrete materials to organize small amounts of data.
- Fill in a symbolic graph based on a concrete or pictorial graph.
- Read simple bar graphs for information.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

The graph shows the kinds of pets people have. How many more people have dogs than have cats? How many people have a dog? How many do not have a pet? What else do you know by looking at the graph?

Rabbit	Dog	Cat	Fish	None
X	X	X	X	X
	X	X	X	X
	X	X		X
	X	X		X
	X			X
	X			
	X			

Standard 5. Problem Solving and Mathematical Reasoning

Students solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade one who meet the standard will:

- Use concepts and skills acquired through previous activities.
- Explain what materials to use, when possible.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade one who meet the standard will:

- Find a solution that is correct and makes sense.
- Tell what their solution is and how they know it works.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade one who meet the standard will:

- Show the solution with another material or in another way when directed to do so.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Each person at your table needs two stickers. Figure out how many stickers your table needs.
2. What is a fair way to share the box of crayons?

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of mathematical communications of others.

For example, students in grade one who meet the standard will:

- Use appropriate mathematical vocabulary; for example, words for simple shapes, attributes, and numbers.
- Show ideas by drawing; by using words and numbers; by building with a variety of concrete materials, such as connecting cubes, pattern blocks, buttons, beads, and color tiles; and by pasting paper representations of materials.
- Explain strategies used in solving problems.
- Share ideas—responding orally, when probed by the teacher, and in writing and drawing (even though responses may lack some coherence or organization).
- Understand oral directions for appropriate mathematical activities.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

Use oral language and physical materials to show how to take numbers apart in order to solve problems when using mental mathematics. For example, a student might say, “Five plus five is ten, and one more makes 11” or “I’ll take three from these four and put it with the seven. Then I have ten, and one more that I have left makes eleven.”

Grade Two

Standard 1. **Number and Operations**

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in number and operations.

For example, students in grade two who meet the standard will:

- Count and write numbers up to 1,000.
- Estimate the amount and count up to 100 objects by groups (e.g., twos, fives, tens).
- Mentally add the numbers 1, 2, 5, and 10 to numbers less than 100; and mentally subtract the numbers 1, 2, 5, and 10 from numbers less than 100.
- Know basic addition and subtraction facts (sums to 10).
- Use knowledge of place value (tens and ones) to find answers to two-digit addition and subtraction problems.
- Identify wholes, halves, and fourths of a simple shape or small group.
- Analyze problem situations and use addition or subtraction appropriately.
- Identify and give the value of coins to one dollar.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Show a way to group these pennies so that it is easy to see whether you have one dollar.
2. There are 27 children in Ms. Brown's room and 29 in this room. How many students will go on the field trip? Explain how you found the answer.
3. Divide the cookie so that everyone at your table has the same amount. There are three people at table A and four people at table B. At which table do people have bigger pieces? How do you know?
4. Mentally add 45 + 5. What is 10 less than 33? What is 32 + 2?

Standard 2. Geometry and Measurement

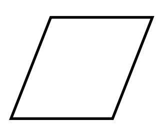
Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade two who meet the standard will:

- Sort and describe shapes by common attributes, such as the number of sides or faces.
- Construct new shapes by putting shapes together or by taking shapes apart.
- Complete and make simple symmetrical designs.
- Measure the length of objects by repeating a unit.
- Use different units to measure the same object and predict whether the measure will be greater or smaller when a different unit is used.
- Solve simple problems involving duration of time.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Make larger squares by putting some square tiles together. How many tiles did the larger square take?
   Draw a picture of the large square you made to show how you put the tiles together.
2. How many ways can you find to cover this shape with pattern blocks?



3. Here is one paper clip. How many paper clips will you need to measure the length of this stick?
4. Measure the length of the table by using two different units. Jody and Jene found that the table was ten new pencils long. Will it take more or fewer tiles to measure the length? Why? What is your estimate of the number of tiles the table measures?

Standard 3. Function and Algebra

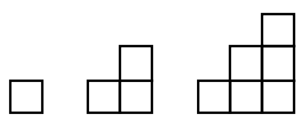
Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in grade two who meet the standard will:

- Identify patterns and extend and build patterns by describing and using the rule for the pattern.
- Identify simple relationships of equality and inequality with the use of concrete materials.
- Use symbols to stand for quantities of concrete materials.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. How many eyes do ten people have? Look at the pattern: one person has two eyes, two people have four eyes, and so forth.
2. Build staircases with blocks as shown below. The first has one step, the second has two steps, and so forth. How many blocks do you need to build the next one?



3. Here is a number sentence that represents all the ways in which you can add two numbers to get ten. What number can you put in the box and what number can you put in the triangle to make the sentence true? Can you find two other numbers? Any more?

$$\square + \triangle = 10$$

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade two who meet the standard will:

- Collect and organize data and display the data in simple graphs and tables.
- Compare data in order to make true statements on simple graphs.
- Classify events as likely or unlikely.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. The graph shows the kinds of pets people have. How many more people have dogs than have cats? Look at the graph and make another statement that you know is true.

Rabbit	Dog	Cat	Fish	None
X	X	X	X	X
	X	X	X	X
	X	X		X
	X	X		X
	X			X
	X			
	X			

2. What are two things that might happen tomorrow? What are two things that probably will not happen tomorrow?

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade two who meet the standard will:

- Use concepts, skills, and strategies acquired through previous activities.
- Explain what materials to use, when possible.
- Draw sketches to model problems.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade two who meet the standard will:

- Find a solution that is correct and makes sense.
- Tell and write their solution and how they know it works.

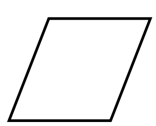
Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade two who meet the standard will:

- Show the solution with another material or in another way when directed to do so.
- Compare the problem with other problems they have solved and tell how it is related.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. The class will make greeting cards, and everyone will need two stickers. How many stickers are needed for the whole class?
2. Divide the cookie so that everyone at your table has the same amount. There are three people at table A and four people at table B. At which table do people have bigger pieces? How do you know?
3. How many ways are there to cover this shape with pattern blocks?



Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of mathematical communications of others.

For example, students in grade two who meet the standard will:

- Use appropriate mathematical terms, vocabulary, and language acquired in prior conceptual work.
- Show ideas in a variety of ways, such as by using drawings, words, numbers, symbols, and simple graphs and tables and by building with a variety of concrete materials.
- Explain strategies used in solving problems.
- Share ideas orally, when probed by the teacher, and in writing and drawing.
- Present ideas appropriately when instructed to respond to a particular audience or for a particular purpose.
- Understand oral and simple written directions for appropriate mathematical activities.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

Use physical materials, diagrams, pictures, and words to show how to take numbers apart in order to solve problems when using mental mathematics. For example, when adding 26 and 45, a student might say, “Twenty plus 40 is 60, and 6 plus 5 is 11, and 60 plus 11 is 71.” Or the student might say, “Twenty-six is like 25 and 1. So 25 and 45 is 70 and plus 1 is 71.”

Grade Three

Standard 1. **Number and Operations**

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in arithmetic and number.

For example, students in grade three who meet the standard will:

- Know addition and subtraction facts (sums to 20).
- Use knowledge of place value to name, write, and show quantities in the hundreds and thousands.
- Add and subtract two- and three-digit numbers.
- Find the answers to simple multiplication problems by using repeated addition, counting by multiples, combining things that come in groups, making arrays, and using area models; and find the answers to simple division problems by putting things into groups and sharing equally.
- Analyze problem situations to figure out when to add, subtract, and multiply.
- Estimate, approximate, or use exact numbers, as appropriate, in real-life situations.
- Find fractions (halves, thirds, fourths) of a whole and of groups of objects.
- Use recall, mental computations, pencil and paper, measuring devices, mathematics texts, manipulatives, calculators, computers, and advice from peers, as appropriate, to achieve solutions.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. What coin combinations will make 75¢?
2. Find the answer to this problem and explain how you solved it: How much money does Sara need to save to have $10 if she already has $6.45?
3. The class will form four teams. How many students will be on each team?

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade three who meet the standard will:

- Describe three-dimensional objects in terms of their two-dimensional faces.
- Describe movement in space by using distance, directions, and turns.
- Estimate and correctly measure the length of common objects in standard units.
- Solve problems involving durations of time.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Give and write simple directions, such as how to go from the classroom to the office.
2. Estimate and measure the length and width of a book and of your desk in centimeters.

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

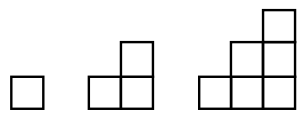
For example, students in grade three who meet the standard will:

- Use a function rule to solve simple problems.
- Recognize an equality relationship and build one with concrete materials; for example, make two lumps of clay equal on a balance scale.
- Use symbols to stand for quantities in simple situations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. How many wheels are on six tricycles? Is there a rule you can use to find out how many wheels are on 11 tricycles?
2. What is the value of your name if A = 1¢, B = 2¢, and so forth?

3. Here are staircases built with blocks. The first staircase has one block, the second staircase has three blocks, and so forth. How many blocks do you need to build a staircase with seven steps? Can you figure it out without building the staircase? Is there a number pattern for building the staircases?



Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade three who meet the standard will:

- Collect and organize data and display the data in diagrams, graphs, charts, and tables.
- Compare data in order to make true statements.
- Make simple inferences about data.
- Describe the situation of certain events that occur more often than others do; for example, red comes up more often on a spinner that is one-half red, one-fourth blue, and one-fourth yellow.
- Find possible combinations and arrangements involving a limited number of variables.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. A survey shows that very few students in grade three bought spaghetti in the school cafeteria on Monday. Does that mean that students in grade three do not like spaghetti? Could there be another explanation?
2. There are three blue blocks in the bag. Can you be sure what color block you will pull out? Add one green block in with the blue. Now can you be sure which color you will get? Why?
3. Tim has three shirts (plaid, striped, and plain) and two pairs of pants (khaki and black) to wear to school. How many different outfits can he wear?

4. Here are two displays of the data gathered in response to the question, “What pets do we have?” What kind of information do you know by looking at the first graph that is not available from the second graph? What information is more obvious from the second graph?

Rabbit	Dog	Cat	Fish	None
X	X	X	X	X
	X	X	X	X
	X	X		X
	X	X		X
	X			X
	X			
	X			

Have a pet	X X X X X X X X X X X X
Do not have a pet	X X X X X

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade three who meet the standard will:

- Use concepts, skills, and strategies acquired through previous activities.
- Use previously learned strategies, skills, knowledge, and concepts to make decisions.
- Use strategies, such as using manipulatives or drawing sketches, to model problems.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade three who meet the standard will:

- Make up and use a variety of strategies and approaches to solve problems and understand approaches that other people use.
- Find a solution that is correct and makes sense.
- Defend their reasoning by explaining how their solution works or how they solved the problem.

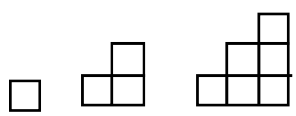
Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade three who meet the standard will:

- Show the solution with another material or in another way.
- Compare the problem to other problems they have solved and tell how it is related.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Here are staircases built with blocks. The first staircase has one block, the second staircase has three blocks, and so forth. How many blocks do you need to build a staircase with seven steps? Can you figure it out without building the staircase? Is there a number pattern for building the staircases?



2. The class will make greeting cards, and each person will need two stickers. Stickers come in packs of eight. How many packs are needed for the whole class?

Standard 6. Mathematical Communication

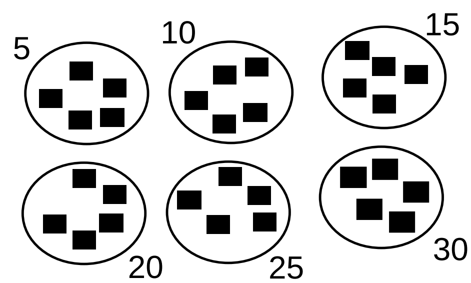
Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of mathematical communications of others.

For example, students in grade three who meet the standard will:

- Use appropriate mathematical terms, vocabulary, and language acquired in prior conceptual work.
- Show ideas in a variety of ways, such as by using words, numbers, symbols, pictures, charts, graphs, tables, and diagrams and by building with a variety of concrete materials.
- Explain strategies used in solving problems and support solutions with evidence in both oral and written form.
- Present ideas appropriately when instructed to respond to a particular audience or for a particular purpose.
- Understand oral and written directions for appropriate mathematical activities.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

Use physical materials, diagrams, pictures, and words to explain how to take numbers apart in order to solve problems when using mental mathematics. For example, when multiplying six by five, a student might say, “My drawing shows 5, 10, 15, 20, 25, 30.”



Or the student might say, “I know that five times five is 25 and one more five makes 30.”

Grade Four

Standard 1. **Number and Operations**

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in number and operations.

For example, students in grade four who meet the standard will:

- Know multiplication facts up to 10 x 10.
- Add and subtract whole numbers.
- Find correct answers to whole-number multiplication and division problems.
- Demonstrate understanding of the place-value system and use this knowledge to solve arithmetic tasks; for example, 36 x 10, 18 x 100, 7 x 1,000, 4,000 ÷ 4.
- Estimate, approximate, round off, or use exact numbers, as appropriate, in calculations.
- Describe and compare quantities by using simple fractions and decimals.
- Describe and compare quantities by using whole numbers.
- Use recall, mental computations, pencil and paper, measuring devices, mathematics texts, manipulatives, calculators, computers, and advice from peers, as appropriate, to solve problems.
- Count and exchange money and solve problems involving addition and subtraction of money.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Estimate and then calculate:
   - ☐ The number of beans in the cup
   - ☐ The number of ceiling tiles in the room
   - ☐ The number of wheels on 37 tricycles
2. Make up a story that goes with 27 x 36 and calculate the answer.

3. Put together fraction pieces (e.g., $^1/_{16}$, $^1/_8$, $^1/_4$, and $^1/_2$) in various ways to make a whole.
4. Organize a budget for a project.
5. Find the answer to this problem and explain how you solved it: The class has saved $85. How much more money does the class need to buy 8 balls at $15 each?
6. Explain how to share 25 as equally as possible in each of these situations:
   - ☐ Four friends share 25 balloons as equally as possible.
   - ☐ Four friends share $25 as equally as possible.
   - ☐ Four friends share 25 cookies as equally as possible.
7. Decide whether to use a calculator, paper and pencil, or mental arithmetic to find the answers to the following problems. Explain why you chose the methods.
   - ☐ 6 x 6,000,000
   - ☐ 13 x 25
   - ☐ 850 ÷ 23
   - ☐ $3. 95 + $0.59 + $22.89

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade four who meet the standard will:

- Identify geometric shapes and use the terms correctly; for example, triangle, square, rectangle, rhombus, parallelogram.
- Identify, classify, and name geometric figures by specific shape properties; for example, symmetry.
- Solve problems by showing relationships between and among figures; for example, by using congruence and similarity and by using transformations, including flips, slides, and rotations.
- Select and use appropriate units for measuring length in both the customary and the metric systems.
- Use models to solve problems related to the perimeter and area of rectangles in simple situations.
- Carry out simple unit conversions; for example, between centimeters and meters and between hours and minutes.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Calculate the areas of rooms when designing a floor plan for a dream house.
2. Find all the shapes you can make with five squares if the sides touch completely.
3. Identify attributes of triangles.
4. Create symmetrical designs with pattern blocks and indicate lines of symmetry.
5. Figure out the approximate area and perimeter of the bottom of a shoe.
6. Order five objects, such as books, rocks, or pumpkins, by weight; do so first by estimating, then by measuring exactly.
7. Use tiles and find the perimeters of all the rectangles that have an area of 24 square units.

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

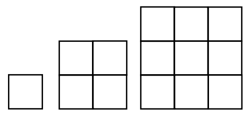
For example, students in grade four who meet the standard will:

- Build iterations of simple numerical (linear and nonlinear) patterns, including multiplicative and squaring patterns, with concrete materials.
- Show that an equality relationship between two quantities remains the same as long as the same change is made to both quantities.
- Use letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations involving concrete materials; that is, students demonstrate an understanding of and use a beginning concept of a variable.
- Use the mathematical symbols +, –, x, ÷, /, $, ¢, %, and . (decimal point) correctly in number sentences and expressions.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Use a table to record functions, such as how many chairs fit at how many tables.

2. Find, make, and describe patterns on the 100 chart; for example, patterns made by multiples of numbers or by additions of 10 to a starting number (4, 14, 24, 34, etc.).
3. Observe and record, in a two-column table, multiplicative patterns with concrete materials; for example, record how many regions are produced by increasing the numbers of folds of paper.
4. Build the fourth, fifth, and sixth iterations in the following sequence:



5. Plot points on a coordinate graph according to the convention that (x, y) refers to the intersection of a given vertical line and a given horizontal line.
6. Use variables to show all the ways of making 10 by adding two whole numbers; that is, $x + y = 10$.

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade four who meet the standard will:

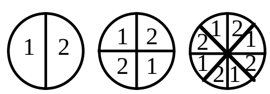
- Collect and organize data and display the data in charts, tables, diagrams, and graphs.
- Use data to answer a question or test a hypothesis.
- Compare data in order to make true statements and draw simple conclusions based on data.
- Gather data to understand the concept of *sample;* for example, that a large sample leads to more reliable information.
- Predict and find out why some outcomes are more likely, less likely, or equally likely to occur.
- Find all possible combinations and arrangements involving a limited number of variables.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Generate a survey question (e.g., How many people are in your family? What is your hair color? What is your favorite pizza top-

ping? What do you do at recess?); predict the results; conduct the survey; organize and graph the data; and write true statements about your findings.

2. Identify a need (e.g., the purchase of new library books) and collect data in order to make a recommendation.
3. Conduct a sampling, then make a conjecture (e.g., How many raisins are in a given box taken from a set of boxes? What colors of tiles are in a bag? What fraction of spins will land on a certain space on a given spinner?).
4. Investigate the temperature during the entire school year.
5. Investigate the possible and likely or unlikely outcomes of rolling two number cubes by rolling them and recording the sums.
6. Find and record all combinations of three ice-cream flavors and two toppings.
7. Use a Venn diagram to record the number of students who wore a sweater to school and the number of students who walked to school.
8. Plan and conduct a probability study that compares the results of using three different spinners, such as those shown here:



9. Compare the growth of a set of plants under a variety of conditions; for example, different amounts of water, fertilizer, and duration of and exposure to sunlight.

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade four who meet the standard will:

- Explain decisions about the approach, materials, and strategies to use in solving the problem.

- Use previously learned strategies, skills, knowledge, and concepts to make decisions.
- Select strategies, such as using manipulatives or drawing sketches, to model problems.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade four who meet the standard will:

- Devise and use a variety of strategies and approaches to solve problems and learn approaches that other people use.
- Make connections among concepts in order to solve problems.
- Solve problems in ways that make sense and explain why those ways make sense; for example, defend the reasoning, explain the solution.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade four who meet the standard will:

- Explain a pattern that may be used in similar situations.
- Explain how the problem is similar to other problems the student has solved.
- Explain how the mathematical concept used in the problem is similar to other concepts in mathematics.
- Explain how the solution may be applied to other school subjects and in real-world situations.
- Generalize the solution to form a rule that applies to other circumstances.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. You may use between zero and three stickers on your greeting card. Determine how many stickers your group needs, collect data for the whole class, and give the teacher the shopping list. Figure out the cost of all the stickers (from a given price list).
2. How high would a stack of 1,000,000 pennies be?
3. How many handshakes would there be altogether if five people in a room were to shake hands with each other just once?
4. Suppose the small triangle of a tangram is worth one cent and the parallelogram, which can be made by putting two small triangles together, is worth two cents. What is the value of each of the other pieces of the tangram?

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of mathematical communications of others.

For example, students in grade four who meet the standard will:

- Use appropriate mathematical terms, vocabulary, and language acquired in prior conceptual work.
- Show ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.
- Explain solutions to problems clearly and logically and support the solutions with evidence in both oral and written form.
- Consider the purpose and audience when communicating.
- Comprehend mathematical concepts from reading assignments and from other sources.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Explain to a student in first grade or to a visitor from outer space why $34 + 17 \neq 3417$.
2. Use words, numbers, or diagrams to explain how to take numbers apart in order to solve problems when using mental mathematics. For example, when multiplying 25 by 6, a student might say: "20 x 6 = 120, and 5 x 6 = 30; 120 + 30 = 150." Or the student might say: "25 x 4 = 100, and 25 x 2 = 50. So the answer is 150 because 100 + 50 = 150."
3. Give an oral presentation of a preliminary investigation of the classification of shapes in order to get peer feedback; then revise the classification scheme to make it clearer.
4. Prepare a report, including graphs, charts, and diagrams, on the optimal number and location of recycling containers by using data from the classroom and the entire school.

Grade Four

This section contains examples of several mathematical tasks with samples of student work for each task. Each task is organized as follows:

- *Task* gives the directions for completing the assignment.
- *Mathematics in the Task* identifies the Challenge Standards addressed in the task (usually more than one) and describes the specific mathematics that students will encounter in completing the task.
- *Setting* describes the situation (in-class, homework, etc.) and the time frame in which the sample work was completed.
- *Samples of Student Work* describes the mathematics evident in each student's work.

Taken together, this collection of tasks is *not* intended to be a test or an assessment of the mathematics standards for students in grade four. The tasks do not cover the full range of mathematical concepts that students need to know, understand, and be able to apply. They are merely examples of what might be included in an assessment or given as classroom assignments.

Most of the tasks are quite small; generally, they are assignments that may be done in class, as homework, or over a one-week period. Because most of the work is from only a few classrooms and was not intended as a formal assessment, *this document does not identify performance levels for the tasks*. Teachers from different classrooms, schools, and districts need to work together to establish performance levels for formal assessments that will be used to determine whether a student has met the mathematics standards. No single task can provide sufficient information to assess whether a particular student has met a standard. A collection of work from a student will need to be assessed to make a valid judgment.

The samples of student work included with each task are intended to be illustrative of high-quality work that can be done by students in grade four and may be considered as some evidence that the students are progressing toward meeting the mathematics standards. The work samples are not in each student's handwriting—they were typed into a computer. However, they contain the actual words (and the spelling) and calculations used by the student; and they reflect, as much as possible, the student's drawings, format, and placement of work. Comments about the work describe the mathematical concepts and the approaches the students used as well as any errors they made. Most errors identified in the students' work are minor. The commentaries do not include the way in which the classroom teacher addressed the errors with students. Most tasks include work from different students. The work of a few students is included for a number of tasks. For example, samples of Mike's work (names are fictitious) are shown for several tasks.

The comments about student work in this document are quite detailed; however, teachers are not expected to *write* commentaries similar to these about an individual student's work. Teachers and students are expected to *think* carefully about the mathematics involved in the assignments and tasks they do in their classrooms and the evidence of mathematical knowledge and understanding clearly demonstrated in the student's work. Teachers may want to use the sample tasks and student work shown here as they discuss what the standards mean, how they are reflected in the classroom program, and the way in which they are exemplified in the work students do.

Task: Bricking a Backyard

There once was a man named Rick. He had a backyard of dirt. He wanted to buy some bricks to cover the dirt. Each brick was 6 inches long and 6 inches wide. His backyard was 9 feet by 10 feet. How many bricks will Rick need to buy? *(This task was written by a student in the class.)*

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

The task requires that students understand the concept of area and how a region may be covered with tiles (bricks). Students may connect their understanding of area to the array model for multiplication. Some students may determine the number of bricks needed for a one-foot square, then calculate (using multiplication) the number needed for the entire backyard. Students must recognize that the brick's dimensions are given in inches and the backyard's dimensions in feet and make a correct conversion. Students need to explain their thinking and reasoning.

Setting

This problem was a two-night homework assignment. Students were expected to explain how they determined the answer and why the answer was correct.

Samples of Student Work: Mike and Nigel

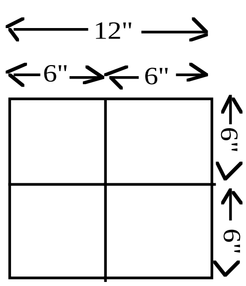
Mike's work includes a diagram labeled to show that four 6-inch bricks cover 1 square foot. The work includes a statement, but does not explain how Mike knew, that the area of the backyard is "9 ft x 10 ft," or "90 of these 1 ft x 1 ft squares," and then the statement that "90 x 4 = 360." His work contains a drawing of a grid (not included in the example) showing the 360 bricks on a 9 by 10 rectangle. The mathematics in the work is correct; and the explanation is brief, clear, and easy to follow.

Nigel's work shows the detailed the process Nigel went through to determine the correct answer. The narrative relates this task to an earlier class activity and includes the generalization that "if I multiply the dimensions I will get the area." The work also relates the backyard to a big array. The work shows the multiplication calculation Nigel used to find the area of the backyard and shows the dimensions labeled with the correct units; that is, *ft.* for the dimensions and *sq. ft.* for the area. It also shows that Nigel then multiplied the area by four and justified the multiplication by showing in a diagram that four bricks "will fit in perfectly" in 1 square foot. There is a labeling mistake in the final calculation, which identifies 360 as *sq. in.* instead of *bricks*. However, in his summary Nigel correctly stated, "My answer is 360 bricks." Nigel's mathematics is correct, and the explanation is clear and easy to follow.

Mike's Work (Bricking a Backyard)

Bricking a Back Yard

I think there are 360 because 4 6" x 6" bricks put together to make a square that is 1 ft x 1 ft.



It would take 90 of these 1 ft x 1 ft squares to cover a 9 ft x 10 ft. backyard.

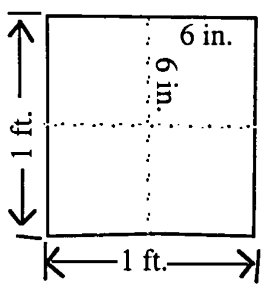
So 90 x 4 = 360.

Nigel’s Work (Bricking a Backyard)

Rick wants to know how many bricks will fit in his yard. His yard is 9 feet by 10 feet and each brick is 6 inches by 6 inches. I'm being asked to find how many bricks Rick needs.

First I found the area of Rick's yard. In an activity before I found that if I multiply the dimensions I will get the area. 90 is the area. I thought of it as being a big array. Then I multiplied the area by 4.

10 ft. dimensions
x 9 ft.
area 90 sq. ft.
x 4
360 sq. in.



I did that because I took a square pretending it was a piece of Ricks yard, and mesured off where the bricks will go and 4 bricks will fit in perfectly.

Another way to figure out this problem is make the big array and count.

This problem might help me in real life by making a brick wall or what Rick is building. It helps me by figuring out how many bricks I need to get.

My answer is 360 bricks.

Task: Symmetrical Quilt

Design a quilt, made of 16 squares, that has at least 4 of the following characteristics:

- Uses at least 2 but no more than 3 geometric shapes
- Has a line of symmetry
- Shows a flip, slide, or turn of a basic component
- Has a repeating pattern
- Shows congruence or similarity of a shape

Draw a picture of the quilt design and accompany it with a detailed explanation of your design and the mathematical ideas it shows.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

Students need to show that they understand and can correctly use geometric terminology. They must design a pattern that shows their understanding of a number of geometric concepts, including symmetry, congruence, similarity, and transformations. They need to communicate their mathematical reasoning.

Setting

This assignment was given as a class assessment at the end of a unit on geometry. Students began their design process in class and had three days to complete the task as homework. Their write-ups were done in class. Students were given an opportunity to revise their work after the teacher's review.

Samples of Student Work: Mike, Babette, and Carmen

Mike's work shows that he has a beginning understanding of geometric concepts that is appropriate for a student in grade four. The work shows that Mike began with a rather complex design for an individual square and repeated it 16 times to form a four-by-four quilt. The basic square is divided by two diagonals into four triangles. The design inside each triangle has two trapezoids, three triangles, and an irregular hexagon, which is referred to as a "six-sided" shape. The triangular design is rotated around the center of the basic square so that each square has bilateral and rotational symmetry.

The triangular design was made by sketching, with no indication of measurements or proportions among the inside shapes. The labeled drawing indicates that Mike recognized that there are two trapezoids even though one of the trapezoids is flipped. There is no indication whether he knows that the six-sided figure is called a hexagon. (A student in grade four is not expected to recognize that the hexagon in this sketch cannot be a regular polygon.)

The work includes a statement, and indicates with arrows in a drawing, that all four triangles (one large and three small) are similar—a statement that may or may not be true. The work does not indicate whether the two small bottom triangles are the same size as the top triangle. (By looking at the drawing and considering the relationships among the figures, one would conclude that the bottom triangles are probably larger.)

In the work the basic square is repeated 16 times. A note indicates that the left side may be flipped to the right. Mike may not be aware that a *flip* indicates a line of symmetry or that the large quilt has all the same lines of symmetry that the basic squares have.

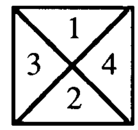
Mike's Work (Symmetrical Quilt)

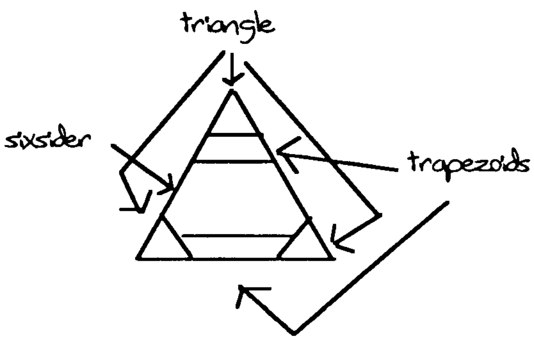
In my class I was asked to make a quilt square and this is how I did mine.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

I have 16 squares in my quilt square.

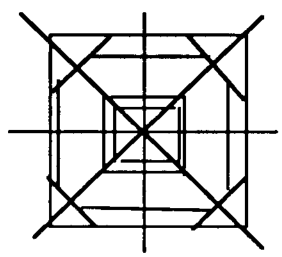
In each square there are 4 triangles.





Inside of each triangle there are 2 trapezoids, 3 triangles and 1 six sided shape.

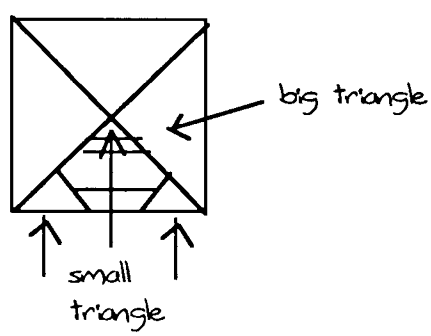
I also have 4 lines of symmetry in each square 1 vertical, 1 horzontal and 2 diagonel



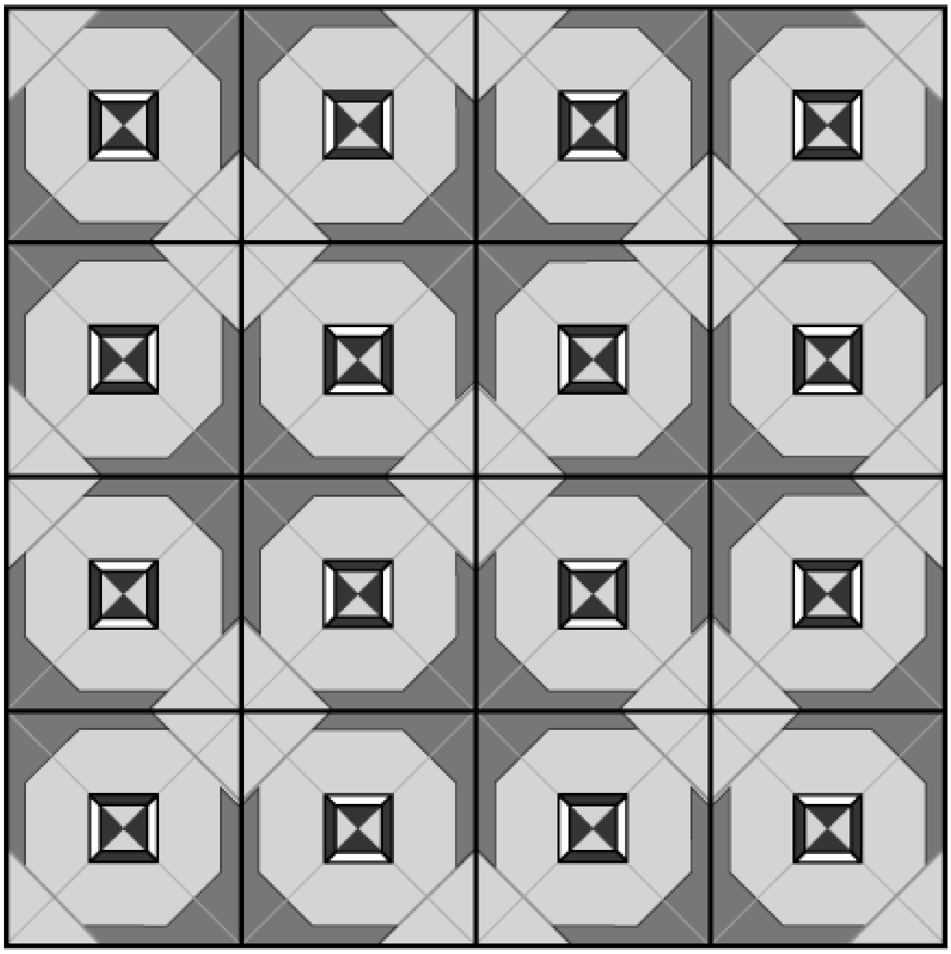
In each of my square there are 4 big triangle in each big triangle there is 3 small triangle. These are similar triangles.

My repeating pattern has one design that goes on and on.

My design goes all over my quilt square.



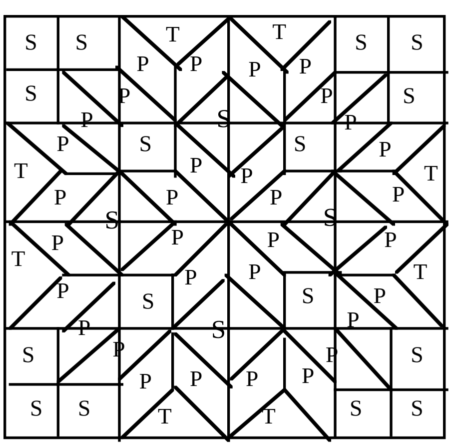
Mike



Babette's work shows a complex design for the quilt that starts in the center with eight parallelograms and radiates symmetrically outward. This design is more complex than those made by many students in grade four. The design uses triangles, squares, and parallelograms as the three building shapes. Many of the pieces (i.e., parallelograms and squares) cross the boundaries of the 16 squares that make up the structure of the quilt. Even though there are some incorrect conclusions in the work, the narrative indicates Babette's understanding of some geometric concepts. For example, the work indicates that Babette understands the concepts of congruence and similarity. However, there is an incorrect conclusion in the statement that two parallelograms "are the same shape but not the same size." That conclusion may have been made because the design was not drawn on squared paper or because the inclusion of the lines of the 16 original squares may have made the size of the parallelograms appear not to be the same. Babette may not understand the difference between flips and turns (reflections and rotations) because she used the word *flip* and stated that the quilt is the same when it is "turned on its side" (a 90° rotation). A note indicates that the quilt has horizontal and diagonal symmetry but there is no mention of vertical symmetry (which may be the *flip* referred to earlier).

Babette's Work (Symmetrical Quilt)

I had to make a math quilt and pick four math things that are in your quilt. Then prove that you do have those four things in your quilt. Here is how I prove that I have symmetry, three geometric shapes, congruent and similar shapes, and that my quilt can be flipped.

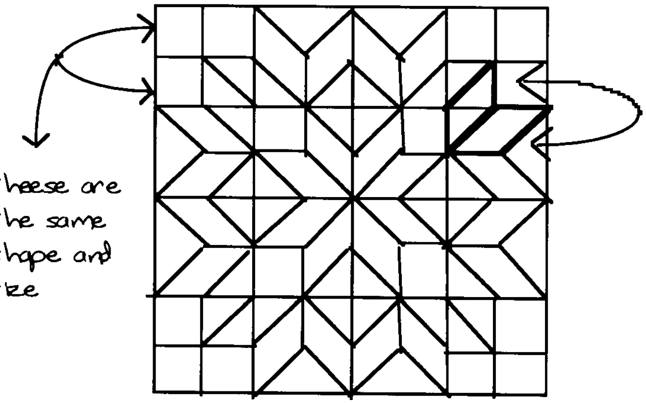


1. I used triangles; parelelagrams, and squares in my math quilt.

T = triangles
S = squares
P= Parellelagrams

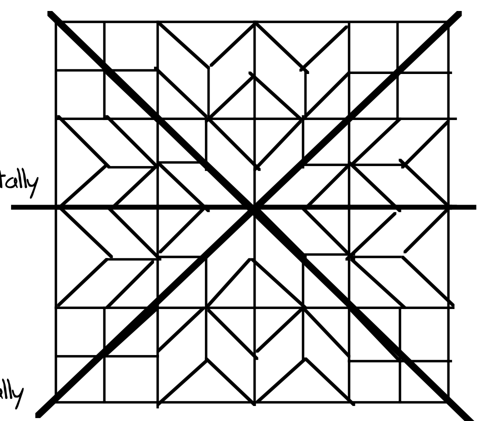
2. There are congruent and similar shapes in my math quilt.

theese are the same shape and size

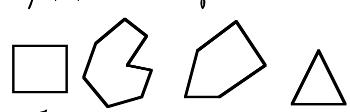


theese are the same shape but not the same size

3. My math quilt is horizontally and diagonally symetrical



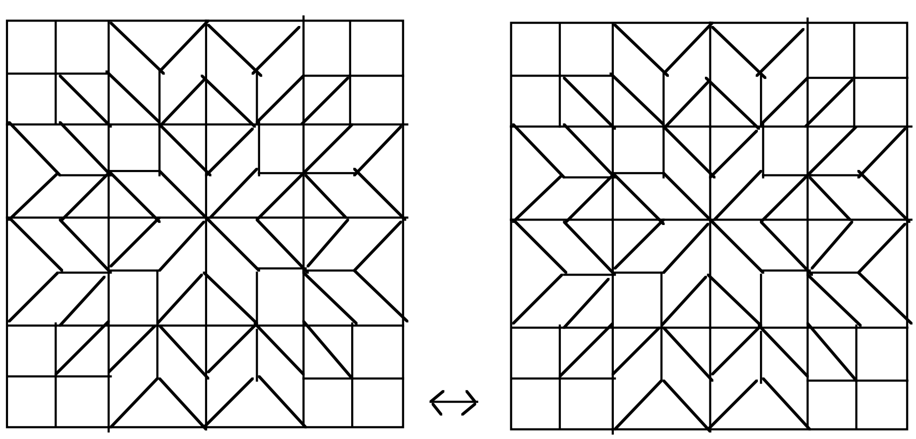
symmetry is when you can cut or fold a shape and all of the parts of the shape are the same shape and size. Here are some symmetrical and not symmetrical shapes.



S N.S N.S S

S = Symmetrical
N.S = Not Symmetrical

4. My math quilt can be fliped



I copyed this from my quilt pattern when it was right side up

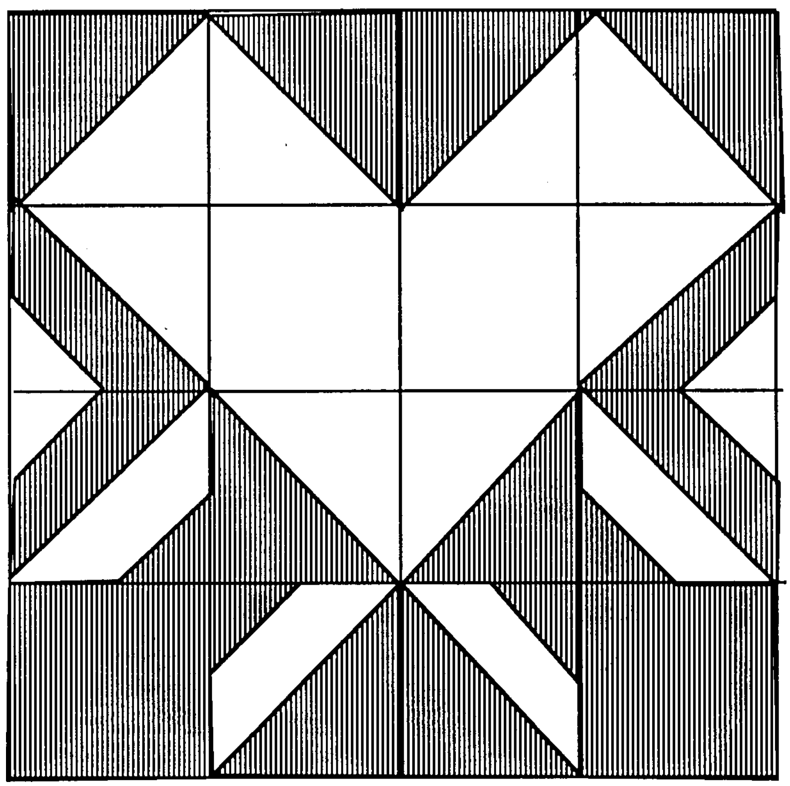
SAME
(congruent)

I copyed this when my quilt is was turned on it's side

Babette

Carmen's quilt shows a fairly simple "heart" design and only a vertical line of symmetry, which Carmen recognizes but does not show. Four of the 16 squares in the quilt are all one color. Six of the squares are cut in half on the diagonal. Triangles, trapezoids, and squares are used to make the design.

Carmen's Work (Symmetrical Quilt)



My quilt square consists of three shapes triangles, trapizoids, and squares, there are no more than three geometric shapes used in this quilt square.

Secondly, it has a line of symetry going verticly in direction.

Thirdly, this quilt square does show a flip of the basic component, but . . . there are separate flip designs for the top and bottom.

The whole quilt contains 4 of the styles we discussed.

I've included a sample quilt square for you to test what I've said.

Task: Sharing Problems

Find and explain the answer for each situation. You may use pictures, diagrams, numbers, and words.

1. A group of children wanted to share a balloon bouquet. There were four children and 29 balloons. How can they do this? How many balloons will each get?
2. Matt, Lupe, James, and Michael found $29.00 on the sidewalk. How can they share this amount? How much money does each boy get?
3. A package of cookies contains 29 cookies. How can Kim, Vivian, Jessica, and Sabrina share them so that each gets the same amount?

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

Each of the three situations involves the division of 29 by 4. However, the way in which the remainder is interpreted is different in each situation; that is, the remainders are a whole number, a decimal (money), and a fraction. Students need to understand the situations, realize that each situation calls for division, find the correct answer, and represent the remainder in a way that is appropriate for the situation.

Setting

This task was a classroom assignment that was completed in less than one class period.

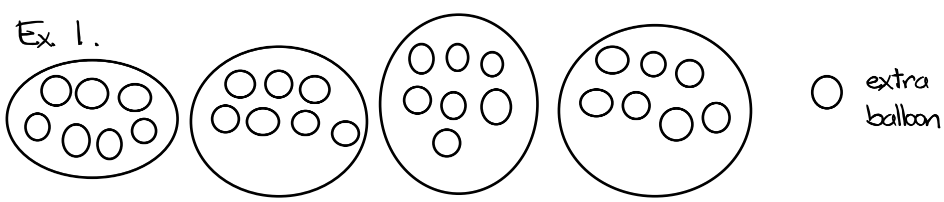
Samples of Student Work: Nigel

Nigel's work shows he has an understanding of the three situations and how each situation can be interpreted as division (e.g., "dividing the 29 balloons into four equal parts"). The work shows the use of a combination of numbers, diagrams, and words. In each situation the remainder is identified and represented in a statement using the appropriate form: whole number, decimal, and fraction.

Nigel's Work (Sharing Problems)

I know 4 children want to share 29 balloons. They can do this by dividing the 29 balloons into for equal parts.

Ex. 1.



4 group. 7 ballons in each group

Ex.2

```
    7 r 1  balloons
4 | 29
   -28
   ---
     1
```

Each child would get 7 ballons and the extra one needs to be poped.

We know 4 boys found $29.00 and want to share it. They can share it by dividing it into 4 equal parts.

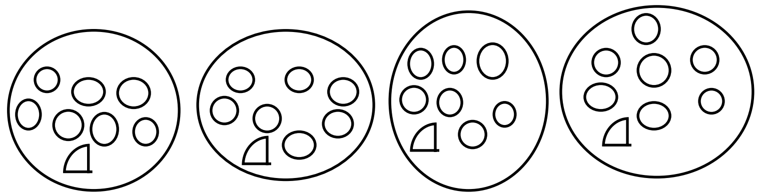
example

```
     7.25
4 | 29.00
   -28
   ---
     10
   -  8
   -----
      20
     -20
     ---
       0
```

Each boy would get $7.25.

We need to divide 29 cookie amongst 4 girls.

ex. 1.



ex.2

$$4 \overline{)29} \quad 7 \text{ r } 1 \text{ cookies}$$
$$\underline{-28}$$
$$1$$

The remaining cookie will be split into fourths. Each girl would get 7 and a 1/4 cookies.

Task: **Hat Sizes**

For the school academic decathlon week, a hat company will make hats for the class but can afford to make them in only three sizes. Measure the heads of students in the class, organize the data, and write a letter to the hat company recommending which three sizes of hats to make. Justify your recommendations in the letter.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

Students need to measure the circumference of their partner's head accurately in centimeters. They need to organize the data from 29 members of the class. They need to base their recommendation on an analysis of the data and clearly communicate the reasons for their recommendations. Some students may indicate a beginning understanding of some statistical terms and concepts by using such words as range, mode, typical, clumps, and so forth in their letters.

Setting

This task was a two-week project that students worked on periodically in class and at home. The project involved working independently, working with a partner, holding class discussions, and conferring periodically with the teacher about their progress. The project began with an in-depth discussion about the task. Each student measured (and double-checked) the circumference of a partner's head, pooled the results in a class list, and copied the list. For homework, students analyzed the data on their own. They discussed their analyses in class; and, finally, wrote letters to the hat manufacturer, summarizing their conclusions.

Samples of Student Work: Connie

Only a part of the student's work on this project is reproduced and commented on here. Students turned in papers for all stages of this project, including the list of student data, their first attempt at organizing the data, and a drawing of the hat.

Connie's work shows the data organized in a chart with each student's name in a list under the appropriate circumference measurement (head size). The statements written below the chart indicate that a column for 58 cm is not included because no one in the class has that measurement. The statements are written in Connie's own language with a few statistical descriptors that were introduced in class *(range, hole, clump)* to talk about the distribution of the data. The statements include generalizations that are not common for students in grade four. For example, approximate fractions are used to identify the number of people in two subgroups compared with the total number in the class—the 15 people with head sizes of 52, 53, and 54 are referred to as "half of the class," and the eight people with a head size of 55 as "about 1/4 of the class." A few of the sentences are not mathematically precise, which is all right for a student in grade four. For example, a statement describing a central tendency reads, "everybody in this class has a headsize around 55 cm."

At the beginning of the letter, Connie recommends that hats be made in sizes 53 cm, 55 cm, and 58 cm and identifies which people with a certain size head could wear which hat; for example, the people with head circumferences of 57 cm and 59 cm could both wear a hat size 58 cm. The number of hats in each size is identified, and one additional hat in each size is ordered in case someone loses a hat or has a change of mind.

Connie's Work (Hat Sizes)

Head Sizes

52 cm.	53 cm.	54 cm.	55 cm.	56 cm.	57 cm.	59 cm.
Amy	Annie	Salim	Armando	Connie	Kate	Jon
Alice	Liz	Franco	Babette	Dan		
Carmen	Ned	Miguel	Carla	Joaquin		
Meg	Nancy	Ebony	Rose	Mike		
Sue	Alan	Maya	Olga			
			Ricardo			
			Troy			
			Nigel			

On this page I didn't put 58 cm. because that was a hole. Nobody in this class has a 58 cm. head. I also noticed that everybody in this class has a headsize around 55 cm. I also noticed that if you add up all the people in 52 cm, 53 cm., 54 cm. (5 people in each one) you'll get 15 people and that's half of this class. On the 55 cm gragh it has 8 people and that's a lot of people, and that's about 1/4 of the class. (8 people X 4 = 32—30 people). I noticed that there is only one person that has 59 cm. and this is Jon, also on 57 cm gragh only. Kate is on that gragh so that means that, that isn't a clump. The range is to 52 cm. to 59 cm. (without 58 cm., a hole).

Dear Hat Company,

We would pick 53, 55, 58 because some people have 52 cm. headsizes. They could fit into 53 cm. hats and 54 cm. headsizes can fit into 55 cm. hats, and 56 cm. heads can almost fit into 55 cm. hats, just a little bit, it will be a little tight, but it will fit, and the 58 cm. hats can fit into 57 cm. and 59 cm. headsizes because on 57 cm. it will fit into 58 cm. because it will be a little loose, but can fit and on 59 cm. it can fit because it's just a little tight. The people who has 56 cm. headsizes can also fit into 58 cm. hats. So a lot of people can fit into 53, 55, and 58 cm. We want 16 53 cm. hats because there is 15 people who fit 53 cm. and just in case somebody loses a hat we added one more hat. We want 13 55 cm hats because there is 12 people who need 55 cm hats and we added 1 more. We also need 3 58 cm. hats because there is 2 people who fit 58 cm. and just in case somebody changes there mind then we added 1 more.

Task: Partner Portrait

Make a drawing of your partner's face. First, using centimeters or inches, estimate the length and width of several facial features of your partner. Then measure those features as accurately as possible and use the measurements to draw a picture of your partner's face. When you have finished, write several sentences about "What I learned."

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

Students need to estimate and measure lengths of facial features in centimeters or inches by using a ruler or tape measure. They need to organize the data and use the measurements to make a life-size drawing of their partner's face. (The drawing is a representation of a three-dimensional face in two dimensions.)

Setting

This task was done in class over several days. Students worked in pairs, each student measuring and making a drawing of his or her partner's face.

Samples of Student Work: Miguel

Miguel's work includes a well-organized and easy-to-read table that shows the estimated and actual measurements of Miguel's partner's face in centimeters. The estimated measurements are close to the actual measurements. Apparently, by holding the ruler in front of him, Miguel was able mentally to transfer a measurement to the real object. He may also have used clues from previous measurements, such as noticing that the width of the mouth is about the same as the width of the eye. The measurements in the table are given to the nearest half-centimeter and are recorded as fractions rather than as decimals, the more conventional notation used in the metric system. In the table all references to length refer to horizontal measurements, and all references to width refer to vertical measurements. The measurements in Miguel's drawing match the measurements recorded in the table, although the "length" of the mouth is slightly off.

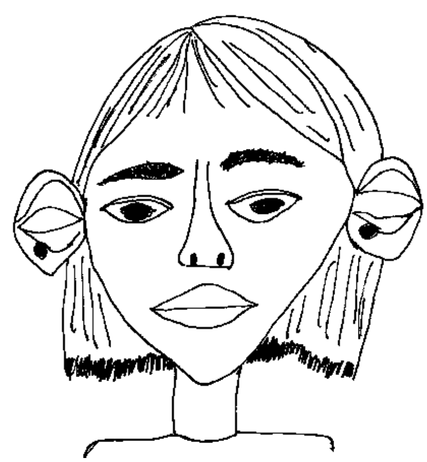
The "What I Learned" statements include several interesting insights. About his drawing (which has been reduced in this document) Miguel wrote, ". . . even though all my measurements were correct, my picture still didn't look like Meg and that means that measurements aren't everything." This statement implies that Miguel checked his measurements ("my measurements were correct") and that he realized that the use of measurements alone was not sufficient to represent a three-dimensional object in two dimensions. (Resolving this dilemma is not expected of students in grade four.) A statement indicates that Miguel used a technique of making "markings" on his picture "so that everything goes in the right place." In addition, he wrote about the need to measure more features than only the "basic parts" if one wanted to include more detail in one's drawing.

Miguel's Work (Partner Portrait)

	Measures	Est/	Act.
1.	Length of eye	3 cm	3cm
2.	width of eye	1 1/2 cm	1 1/2 cm
3.	Length of Nose	2 cm	3 cm
4.	width of Nose	4 1/2 cm	5 cm
5.	Length of mouth	5 cm	5 cm
6.	width of mouth	1 1/2 cm	1 1/2 cm
7.	Length of Face	18 cm	13 cm
8.	width of face	20 cm	18 cm
9.	Length of ears	3 1/2 cm	3 1/2 cm
10.	width of ears	5 cm	5 cm
11.	Length of eyebrow	3 1/2 cm	4 1/2 cm
12.	width of eyebrow	1/2 cm	1/2 cm

What I Learned

I learned how to make a reasonable estimate, as you can see since six of my estimates were correct. I also learned that, even though all my measurements were correct, my picture still didn't look like Meg and that means that measurements aren't everything. Also, I learned how to make markings on my picture so that everything goes in the right place. I learned to only measure the main parts of the person youre drawing so on your picture it only shows the basic part, unless you want detail. The you don't only measure the basic parts. Last of all I learned the area of most of the characteristics that make up my partners face.



Note: The drawing has been reduced for this publication, but the written measurements are actual sizes used in the original drawing.

Grade Five

Standard 1. **Number and Operations**

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in arithmetic and number.

For example, students in grade five who meet the standard will:

- Add, subtract, multiply, and divide whole numbers and decimals and know when to use each operation.
- Explain whether a situation calls for an accurate answer or an estimate and explain the choice of using paper and pencil, mental computation, or a calculator to find the answer.
- Understand and use the inverse relationships between addition and subtraction and between multiplication and division.
- Identify and use equivalent fractions, decimals, and percents for halves, fourths, tenths, and one whole.
- Identify and use factors, multiples, and primes.
- Estimate (e.g., rounding, approximating) or use exact numbers, as appropriate, in calculations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Find two 2-digit numbers whose product is 1,512.
2. Figure out the number of people who represent one-tenth, one-fourth, and one-half of the population of Burb Town, which has a population of 6,200 children and 14,000 adults.
3. How long would it take to say your name one million times? How long would it take to count to a million?
4. Our class of 30 is raising money to go to the amusement park. Individual tickets cost $5.75. We are selling ice-cream bars for 85¢ each and will make a profit of 15¢ on each one. How many ice-cream bars do we need to sell to raise enough money for our whole class to go to the park?

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade five who meet the standard will:

- Identify and describe properties of assorted two- and three-dimensional figures, including squares, triangles, other polygons (e.g., pentagons, hexagons, octagons), circles, cubes, and rectangular prisms (boxes).
- Analyze and generalize geometric patterns, such as tessellations and sequences of shapes.
- Model situations geometrically to formulate and solve problems.
- Identify and plot points in the first quadrant of the coordinate plane.
- Choose appropriate units (customary or metric), measure lengths, and informally find areas of geometric figures.
- Explain the differences between length and area; that is, between linear and square units.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Find the length and width of a rectangle whose sides are two-digit numbers and whose area is 1,512 square centimeters.
2. Examine logos of businesses in the yellow pages for rotational and bilateral symmetry.
3. Use tiles to investigate the relationship between area and perimeter. Arrange 24 tiles so that each tile has at least one side fully touching a side of another tile. What is the perimeter of the arrangement? Find the largest and smallest perimeters that can be made for different arrangements. Do the same using 25 tiles, 30 tiles, and so forth. What do you notice about the shapes of the arrangements as the perimeter changes?
4. Make a two-dimensional cardboard or paper replica of oneself, using measurements for lengths and widths of body parts that are half the size of one's own body.

Standard 3. Function and Algebra

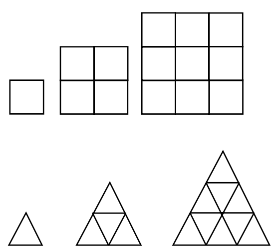
Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in grade five who meet the standard will:

- Discover, generalize, and informally describe linear and nonlinear patterns.
- Represent simple numerical relationships in tables, graphs in the coordinate plane, and verbal or symbolic rules.
- Analyze tables and graphs to determine simple functional relationships.
- Find solutions for unknown quantities in simple equations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

Continue both patterns with blocks and record in a table the number of blocks needed to build each pattern.



Pattern	# of blocks
1	1
2	4
3	9

Find the sum of the first two consecutive odd numbers, the first three, the first four, and so forth. Why are all three patterns the same? Why is this pattern called “square numbers”?

Standard 4. Statistics and Probability

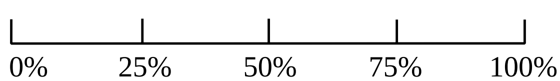
Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade five who meet the standard will:

- Collect and organize data and display and interpret data in appropriate tables, charts, and graphs.
- Describe informally how the data are dispersed, such as whether they are evenly distributed, have one or more modes, have one or more outliers.
- Make conclusions and recommendations based on data analysis and critique the conclusions and recommendations of others.
- Identify characteristics that make a population sample representative or biased.
- Make predictions for simple probability situations.
- Express outcomes of experimental probability situations verbally and numerically.
- Represent possible outcomes for probability situations in an organized way (e.g., in tables, grids, or tree diagrams).

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Gather and analyze data from the neighborhood and compare the data with published statistics for the city, state, or nation.
2. How many of you were born in California? If you were born in this state, is it likely that your parents were too? What about your grandparents? Do you think the results would be similar if students from another state were asked the same questions about their state?
3. Estimate the likelihood of future events on this scale:



   a. All our classmates will be here tomorrow.
   b. A live elephant will visit our classroom this year.
   c. It will rain tomorrow. It will rain this year.
   d. I will drink some milk today.

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade five who meet the standard will:

- Formulate and solve a variety of meaningful problems.
- Extract pertinent information from situations and figures and identify what additional information is needed.
- Formulate conjectures and argue, short of formal proof, why they must be or seem to be true.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade five who meet the standard will:

- Use and invent a variety of approaches and understand and evaluate the approaches of others.
- Employ problem-solving strategies, such as illustrating the problem with sketches to make sense of complex situations or organizing information in a table.
- Explain, when helpful, how to break a problem into simpler parts.
- Solve problems for unknown or undecided quantities by using algebra, graphs, sound reasoning, and other strategies.
- Integrate concepts and techniques from different areas of mathematics.
- Make sensible, reasonable estimates.
- Make justified, logical statements.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade five who meet the standard will:

- Verify and interpret results in the context of the original problem.
- Generalize solutions and strategies to address new problem situations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. How long would it take to say your name one million times? How long would it take to count to a million?
2. Use tiles to investigate the relationship between area and perimeter. Arrange 24 tiles so that each tile has at least one side fully touching a side of another tile. What is the perimeter of the arrangement? Find the largest and smallest perimeters that can be made for different arrangements. Do the same using 25 tiles, 30 tiles, and so forth. What do you notice about the shapes of the arrangements as the perimeter changes?
3. How much pizza should be ordered for a class party? Explain your reasoning.
4. You plan to build a pen with at least one gate for your dog. Fencing is $1 per foot, fence posts are $2 each, and gates are $5 each. You can afford only $40 altogether. How would you design your pen in order to get the greatest possible area? Explain what you considered in making your plan.

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate understanding of mathematical communications of others.

For example, students in grade five who meet the standard will:

- Use mathematical language and representations with appropriate accuracy; for example, in formulating numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams.
- Organize work, explain facets of a solution orally and in writing, label drawings, and use other techniques to make meaning clear to the audience.
- Use mathematical language to make complex situations easier to understand.
- Exhibit developing reasoning abilities by justifying statements and defending work.
- Show understanding of concepts by explaining ideas not only to teachers but also to fellow students or younger children.
- Comprehend mathematical concepts from reading assignments and from other sources.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. How much pizza should be ordered for a class party? Explain your reasoning.
2. You plan to build a pen with at least one gate for your dog. Fencing is $1 per foot, fence posts are $2 each, and gates are $5 each. You can afford only $40 altogether. How would you design your pen in order to get the greatest possible area? Explain what you considered in making your plan.
3. A family of six ordered 13 hot dogs. Nine hot dogs had mustard; three had catsup; eight had relish; four had both mustard and relish; and three had mustard, catsup, and relish. Figure out how many hot dogs had no mustard, relish, or catsup (nothing) on them. You may use drawings in your explanation.

Grade Six

Standard 1. Number and Operations

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in arithmetic and number.

For example, students in grade six who meet the standard will:

- Add, subtract, multiply, and divide whole numbers and decimals consistently and accurately.
- Identify and use equivalent fractions, decimals, and percents for common fractions, such as halves, thirds, fourths, fifths, eighths, and tenths.
- Compare the size of and order fractions, decimals, and percents for common fractions, such as halves, thirds, fourths, fifths, eighths, and tenths.
- Interpret *percent* as a part of 100 and as a means of comparing quantities of different sizes or changing sizes.
- Understand the inverse relationships between addition and subtraction and between multiplication and division and use these relationships in solving problems.
- Use integers (positive and negative whole numbers) in common situations.
- Estimate (e.g., rounding, approximating) or use exact numbers, as appropriate, in calculations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Locate 0.05, 0.1, 6, 1/4, 2/3, –0.5, and 2.33 on a number line.
2. The population of Burb Town doubles every year. In 1995 the population was 512. What will the population be in the year 2000? What was the population in the year 1992?
3. Figure out how to compute a tip of 10%, 15%, or 20% other than by multiplying an amount by 0.1, 0.15, or 0.2.

4. Take your pulse for 15 seconds. According to this measurement, how many times would you say your pulse beat since you got up this morning? This week? Since you were born? Do you think your answers are exact? Why or why not?
5. George said that if he won the lottery, he would share it with his friends. He would keep 1/3, give 1/4 to Diane, and give 1/5 to Tony. Is this plan possible? If so, will there be an amount left? If not, why not?

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade six who meet the standard will:

- Identify triangles (e.g., isosceles, obtuse) and describe their properties.
- Analyze and generalize geometric patterns, such as tessellations and sequences of shapes.
- Model situations geometrically to formulate and solve problems.
- Develop and use formulas for the area and perimeter of rectangles.
- Identify and plot points in the coordinate plane.
- Choose appropriate units of linear measure and convert with ease between units (e.g., between inches and yards) within the customary or metric system. (Note that conversions between customary and metric units are not required.)
- Use proportional relationships to interpret distances on maps and to make scale drawings (enlargements and reductions).

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. If the length of a rectangle doubles and the width triples, what happens to the area of the rectangle?
2. Investigate the area around the school and the neighborhood to describe the size of an acre and of a square mile.
3. Draw an enlargement or a reduction of a cartoon, poster, map of school or home, or similar item accurately to scale.
4. Can you draw the following figures?

   - ☐ Rectangle with four sides of equal length
   - ☐ Parallelogram with four right angles

- ☐ Parallelogram with four sides of equal length and no right angles
- ☐ Parallelogram with four sides of equal length and four right angles

5. Design a recreational area for a one-acre site. Staying within a limited budget, recommend equipment for the area and determine the cost of the equipment.

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in grade six who meet the standard will:

- Discover, describe, and generalize patterns, including linear and nonlinear relationships, and represent them with variables and expressions.
- Represent linear and nonlinear relationships in tables, graphs in the coordinate plane, and verbal or symbolic rules.
- Analyze tables and graphs to determine simple functional relationships.
- Find solutions for unknown quantities in simple equations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Use a diagram, a table, a graph, words, a formula, or a combination of these methods to show the relationships between the length of the sides of a square and (a) the perimeter of the square; and (b) the area of the square.
2. Your principal wants to hire you to work for her for ten days. She will pay you either $6 each day for all ten days; or $1 the first day, $2 the second day, $3 the third day, and so on; or $0.10 the first day and for each day thereafter twice the amount of the day before. In which way would you earn the most money? In which way would you earn the least money?

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade six who meet the standard will:

- Collect and organize data and display data in appropriate tables; charts; and graphs, including bar, line, and simple circle graphs.
- Find and interpret measures of central tendencies of data, using mean and median.
- Make conclusions and recommendations based on data analysis and critique the conclusions and recommendations of others.
- Describe the sample population and discuss whether the data collected are a representative sample.
- Demonstrate recognition that a situation may have two or more equally likely results.
- Make predictions based on experimental or theoretical probabilities.
- Predict the result of a series of trials once the probability for one trial is known.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Bring in newspaper articles containing graphs or charts of data and conclusions based on the data. Interpret the graphs or charts. Which conclusions are plausible? Possible? Impossible? What other questions would you ask to obtain data that would support or negate your reasoning?
2. Explain whether it is the most advantageous to use three tetrahedra, two cubes, or one dodecahedron to arrive at a specific number when rolling polyhedral dice.

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade six who meet the standard will:

- Formulate and solve a variety of meaningful problems.
- Extract pertinent information from situations and figures and identify what additional information is needed.
- Formulate conjectures and argue, short of formal proof, why they must be or seem to be true.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade six who meet the standard will:

- Use and invent a variety of approaches and understand and evaluate the approaches of others.
- Invoke problem-solving strategies, such as illustrating problems with sketches to help make sense of complex situations or organizing information in a table.
- Explain, when helpful, how to break a problem into simpler parts.
- Solve problems for unknown or undecided quantities by using algebra, graphs, sound reasoning, and other strategies.
- Integrate concepts and techniques from different areas of mathematics.
- Make sensible, reasonable estimates.
- Make justified, logical statements.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade six who meet the standard will:

- Verify and interpret results in the context of the original problem.
- Generalize solutions and strategies to address new problem situations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Find two numbers larger than 10 that can be multiplied together to give the answer 708. Explain how you decided on those numbers.
2. Your principal wants to hire you to work for her for ten days. She will pay you either $6 each day for all ten days; or $1 the first day, $2 the second day, $3 the third day, and so on; or $0.10 the first day and for each day thereafter twice the amount of the day before. In which way would you earn the most money? In which way would you earn the least money?

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of the mathematical communications of others.

For example, students in grade six who meet the standard will:

- Use mathematical language and representations with appropriate accuracy; for example, in formulating numerical tables and equations, simple algebraic equations, formulas, charts, graphs, and diagrams.
- Organize work, explain facets of a solution orally and in writing, label drawings, and use other techniques to make meaning clear to the audience.
- Use mathematical language to make complex situations easier to understand.
- Exhibit developing reasoning abilities by justifying statements and defending work.
- Demonstrate understanding of concepts by explaining ideas not only to teachers but also to fellow students or younger children.
- Demonstrate comprehension of mathematical concepts from reading assignments and from other sources.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Can you draw the following figures?
   - ☐ Rectangle with four sides of equal length
   - ☐ Parallelogram with four right angles

- ☐ Parallelogram with four sides of equal length and no right angles
- ☐ Parallelogram with four sides of equal length and four right angles

2. A friend of yours has just moved to the United States and must ride the bus to and from school each day. The bus ride costs 50 cents. Your friend must have exact change and must use only nickels, dimes, and quarters. Your friend has a problem because she does not yet understand U.S. money and does not know how to count the money.

a. Help your friend find the right coins to give to the bus driver. Draw and write something on a sheet of paper that will help her. She needs a sheet of paper showing which combinations of coins may be used to pay for the 50-cent bus ride.

b. Be sure to organize your paper so that the information is clear and helpful to your friend.

Grade Seven

Standard 1. Number and Operations

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in arithmetic and number.

For example, students in grade seven who meet the standard will:

- Add, subtract, multiply, and divide rational numbers consistently and accurately.
- Understand the inverse relationships between addition and subtraction and between multiplication and division and use the inverse operation to determine unknown quantities in equations.
- Identify and use equivalent fractions, decimals, and percents.
- Compare the size of and order fractions, decimals, and percents.
- Represent and compare the size of large and small numbers by using place value and exponents.
- Compare the size and order of integers.
- Demonstrate familiarity with characteristics of operations and numbers (e.g., divisibility, prime factorization) and with principles of rational numbers (e.g., commutativity and associativity), short of formal statements.
- Estimate (e.g., rounding, approximating) or use exact numbers, as appropriate, in calculations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. On an allowance of $12 a week, which is better: a $3 increase or a 25% increase in the allowance?
2. What determines whether you report information in fractions, percents, or decimals? If you knew that 271 of 952 students rode bikes to school, would you say about 1/4 of the people, 25% of the people, .25 of the people, or about 300 people rode bikes? Would you report the fraction 271/952 of the people?

3. Compare the numbers one million and one billion. If you had a million dollars in thousand-dollar bills, the stack would be 4 inches high. How high would the stack be if you had a billion dollars' worth of thousand-dollar bills? Could you carry it?
4. The same kind of chair is on sale at two stores. How could a store advertise a discount of 30 percent and still make more money than the store down the street makes by selling the chair at a discount of 20 percent?
5. Analyze advertisements for different music clubs, decide which club offers the best value for the money, and justify your decision.

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade seven who meet the standard will:

- Identify and describe the properties of and relationships among quadrilaterals (e.g., rhombus, square, rectangle, parallelogram, trapezoid).
- Identify rotational and bilateral symmetry in two-dimensional shapes.
- Use proportional relationships to demonstrate that figures are similar.
- Analyze and generalize geometric patterns, such as tessellations and sequences of shapes.
- Model situations geometrically to formulate and solve problems.
- Develop and use formulas for the area of triangles.
- Analyze and describe the relationship between the area and the perimeter of two-dimensional figures (e.g., consider what happens to the area when the perimeter of a figure is doubled).

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. What is the sum of the interior angles of a triangle? How can you prove it? What is the sum of the interior angles of a rectangle? Of any quadrilateral? Show how to find the sum of the interior angles of any polygon.
2. Display data in an accurately drawn and divided pie chart.

3. Find the minimum perimeter of a rectangle with an area of 25 square units if the dimensions are whole numbers. Repeat the procedure for rectangles with areas of 30, 35, and 40 square units. Figure out how to predict the dimensions of the rectangle with the minimum perimeter if you continue this investigation with larger whole numbers.
4. Describe the size of one million of an object, such as a shoe box, a penny, a pack of notebook paper. Describe the size of one-millionth of that same object.
5. Use paper models of triangles and parallelograms to show how the formulas for finding their areas can be derived.
6. Build larger and larger cubes from unit cubes. Use tables to show how the volume and the surface areas increase. Then suppose that the outside of each constructed cube is painted. If a large cube is taken apart, how many of the unit cubes (1 x 1 x 1) are painted on three faces, on two faces, on one face, and on zero faces? Is there a pattern in the number of faces painted?

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in grade seven who meet the standard will:

- Discover, describe, and generalize patterns, including linear and simple exponential relationships, and represent them with variables and expressions.
- Represent relationships in tables, graphs in the coordinate plane, and verbal or symbolic rules.
- Analyze tables, graphs, and rules to determine functional relationships.
- Find solutions for unknown quantities in simple equations and inequalities.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Graph and explain the growth of a population of organisms that doubles in number once a day.
2. Compare the growth of a set of plants under a variety of conditions; for example, by varying the amounts of water, the use or absence of fertilizer, and the duration of and exposure to sunlight.

3. Build larger and larger cubes from unit cubes. Use tables to show how the volume and the surface areas increase. Find a rule to describe how each measure increases. Then suppose that the outside of each constructed cube is painted. If a large cube is taken apart, how many of the unit cubes (1 x 1 x 1) are painted on three faces, on two faces, on one face, and on zero faces? Are there patterns in the number of faces painted? Can you find rules or equations to describe the patterns?

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade seven who meet the standard will:

- Organize data in charts and graphs, including scatter plots; bar, line, and circle graphs; and Venn diagrams.
- Collect and organize data and display data in appropriate tables, charts, and graphs.
- Analyze data for frequency and distribution, including mode and range.
- Analyze central tendencies of data appropriately, using mean and median.
- Make conclusions and recommendations based on data analysis.
- Critique the conclusions and recommendations derived from others' statistics.
- Demonstrate an understanding of the effects on reliability of sampling procedures and of missing or incorrect information.
- Formulate hypotheses to answer a question and use data to test the hypotheses.
- Demonstrate an understanding of why two results are equally likely; construct sample spaces; and determine probabilities of events.
- Make predictions based on experimental or theoretical probabilities.
- Predict the result of a series of trials once the probability for one trial is known.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Develop and analyze a game of chance for a school carnival.
2. Display the data collected from a survey in an accurately drawn and divided pie chart.

3. A random “favorite fruit” poll was taken in the Elmherst Junior High School cafeteria. Each student polled could vote for only one kind of fruit. The chart shows the results of the poll. If there are 600 students in the school, approximately how many should be expected to prefer apples?

Favorite Fruit	*Votes*
Apples	13
Oranges	8
Bananas	5
Grapes	4

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade seven who meet the standard will:

- Formulate and solve a variety of meaningful problems.
- Extract pertinent information from situations and figures and identify what additional information is needed.
- Formulate conjectures and argue, short of formal proof, why they must be or seem to be true.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade seven who meet the standard will:

- Use and invent a variety of approaches and understand and evaluate the approaches of others.
- Invoke problem-solving strategies, such as illustrating the problem with sketches to clarify the situation or organizing information in a table.
- Explain, when helpful, how to break a problem into simpler parts.
- Solve problems for unknown or undecided quantities by using algebra, graphs, sound reasoning, and other strategies.

- Integrate concepts and techniques from different areas of mathematics.
- Make sensible and reasonable estimates.
- Make justified and logical statements.

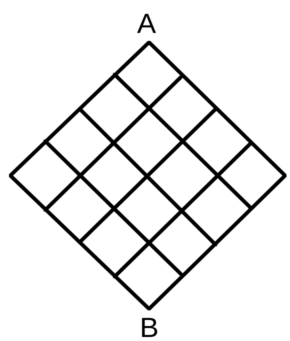
Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade seven who meet the standard will:

- Verify and interpret results in the context of the original problem.
- Generalize solutions and strategies to address new problem situations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. How many downward paths can you find from A to B along the lines of the grid?



2. Each cereal box contains a baseball card. There are six different cards in a set. How many boxes of cereal must you purchase to get all six cards?
3. How many three-digit numbers can you create, using digits one through nine, if the digits are placed in descending order? An example of a number in descending order is 741. The digits in 682 are not in descending order.
4. Jessie collected $1.60 from each customer on her paper route. Mr. Hendron paid her in nickels, dimes, and quarters. He gave her 19 coins in all. How many of each kind of coin could he have given her?
5. Build larger and larger cubes from unit cubes. Use tables to show how the volume and the surface areas increase. Find a rule to

describe how each measure increases. Then suppose that the outside of each constructed cube is painted. If a large cube is taken apart, how many of the unit cubes (1 x 1 x 1) are painted on three faces, on two faces, on one face, and on zero faces? Are there patterns in the number of faces painted? Can you find rules or equations to describe the patterns?

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of the mathematical communications of others.

For example, students in grade seven who meet the standard will:

- Use mathematical language and representations with appropriate accuracy; for example, in formulating numerical tables and equations, simple algebraic equations, formulas, charts, graphs, and diagrams.
- Organize work, explain facets of a solution orally and in writing, label drawings, and use other techniques to make meaning clear to the audience.
- Use mathematical language to make complex situations easier to understand.
- Exhibit developing reasoning abilities by justifying statements and defending work.
- Demonstrate understanding of concepts by explaining ideas not only to teachers but also to fellow students or younger children.
- Demonstrate comprehension of mathematical concepts from reading assignments and from other sources.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. A friend says he is thinking of a number. When 100 is divided by the number, the answer is between one and two. Give at least three statements that must be true of the number. Explain your reasoning.
2. Luke wants to paint one wall of his room. The wall is eight meters wide and three meters high. It takes one can of paint to cover 12 square meters, and the paint costs $9 for two cans. What else does Luke need to consider? Write a plan for an oral presentation for this painting job.

3. The coach and nine children from the baseball team go out for Sloshees after the game. The convenience store has only two sizes of cups left: the small Sloshee for 50¢ and the large for $1. The coach is dieting and does not want anything. He has only $6 to spend on the team. How many Sloshees of each size can the coach buy for the team members? Make a table showing a list of possible solutions. Explain how each solution best solves the problem of what the coach can buy.

Grade Eight

Standard 1. Number and Operations

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in arithmetic and number.

For example, students in grade eight who meet the standard will:

- Add, subtract, multiply, and divide rational numbers consistently and accurately and raise rational numbers to whole number powers.
- Understand the inverse relationships between addition and subtraction, multiplication and division, and exponentiation and root-extraction and use the inverse operation to determine unknown quantities in equations.
- Use and convert the different kinds and forms of rational numbers (e.g., fractions, decimals, percents, integers) consistently and accurately.
- Demonstrate familiarity with characteristics of operations and numbers (e.g., divisibility, prime factorization) and with principles of rational numbers (e.g., commutativity and associativity), short of formal statements.
- Demonstrate understanding of proportion by solving problems involving equivalent fractions or equal ratios.
- Order numbers by using the > and < relationships and by locating numbers on a number line and have a sense of the magnitude and relative magnitude of numbers. (Note that scientific notation is not required.)
- Represent and compare the size of large and small numbers by using place value, exponents, and scientific notation.
- Demonstrate an understanding of the concept of density of numbers by finding a number between two rational numbers and approximating a decimal value for irrational numbers (e.g., finding the length of the side of a square with an area of two).
- Estimate (e.g., rounding, approximating) or use exact numbers, as appropriate, in calculations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Figure out the additional percent of beef needed to make a one-third-pound burger into a one-half-pound burger.
2. In a school with 1,000 lockers, one student opens every locker, another student closes every other locker (second, fourth, sixth, etc.), a third student changes every third locker (opens closed lockers and closes open lockers), and so forth, until the thousandth student changes the thousandth locker. At that point, which lockers are open? Why?
3. Investigate and describe the relationships among and the properties of the numbers in Pascal's triangles.
4. Is the sum of two consecutive integers odd or even? Always? How about the product of two consecutive numbers? Why? What can you say about three consecutive numbers?
5. Find the last two digits of 6^{1000}.
6. Research the national debt. How high would a stack of bills have to be to match the size of the debt?

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in grade eight who meet the standard will:

- Identify and use geometric terms correctly, including the terms diameter, radius, circumference, parallel, perpendicular, angle, plane, congruent, similar, and symmetry.
- Analyze and generalize geometric patterns, such as tesellations and sequences of shapes.
- Model situations geometrically to formulate and solve problems.
- Translate, rotate, and reflect polygons on the coordinate plane and describe the transformations by identifying corresponding points.
- Identify appropriate units and use and read measurement tools (e.g., ruler, protractor, thermometer) for determining length, area, volume, angle, weight, capacity, time, and temperature.
- Choose appropriate units and measure length, area, and volume of geometric figures; and explain the differences between corresponding linear units, square units, and cubic units.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Study the steepness of wheelchair ramps and stairs.
2. Henry wanted to make a box from an 8-inch by 14-inch piece of cardboard. He plans to cut squares from each corner and tape the corners. What are the dimensions of the box that will contain the largest volume?
3. What is the relationship between a 10-inch pizza and a 14-inch pizza? How much more pizza do you get when you buy a 14-inch pizza than when you buy a 10-inch pizza? If the 10-inch pizza costs $7.75, what would be a fair price for the 14-inch pizza?

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in grade eight who meet the standard will:

- Discover, describe, and generalize patterns and functions, including linear, exponential, and simple quadratic relationships, and represent them as algebraic equations.
- Represent the same function in several ways: graph, set of ordered pairs, algebraic rule, written statement.
- Analyze tables, graphs, and rules to determine functional relationships.
- Solve linear equations and inequalities in one variable.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Study the steepness of wheelchair ramps and stairs.
2. Bricklayers use the rule $N = 7 \cdot L \cdot H$ to determine the number (N) of bricks needed to build a wall L feet long and H feet high. Examine a brick wall or a portion of a brick wall to see whether this rule seems to be true. If it works, why? If not, what would be a better formula?

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in grade eight who meet the standard will:

- Organize data in charts and graphs, including scatter plots; bar, line, and circle graphs; and Venn diagrams.
- Collect and organize data and display data in appropriate tables, charts, and graphs.
- Analyze data for frequency and distribution, including mode and range.
- Analyze central tendencies of data appropriately, using mean and median.
- Make conclusions and recommendations based on data analysis.
- Critique the conclusions and recommendations derived from others' statistics.
- Explain the effects on reliability of sampling procedures and of missing or incorrect information.
- Formulate hypotheses to answer a question and use data to test the hypotheses.
- Demonstrate a recognition that two or more results are equally likely; construct sample spaces; and determine probabilities of events.
- Make predictions based on experimental or theoretical probabilities.
- Predict the result of a series of trials once the probability for one trial is known.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Use box-and-whiskers plots, stem-and-leaf plots, and bar graphs to compare the characteristics of the boys and girls in the class. Compare the kinds of information provided by the different displays.
2. Your cousin in grade five is convinced that the probability of rolling a 12 on two numbered cubes is 1/11. Explain to your cousin why this is incorrect and convince your cousin of the actual probability of getting 12.

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in grade eight who meet the standard will:

- Formulate and solve a variety of meaningful problems.
- Extract pertinent information from situations and figures and identify what additional information is needed.
- Formulate conjectures and argue, short of formal proof, why they must be or seem to be true.

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in grade eight who meet the standard will:

- Use and invent a variety of approaches and understand and evaluate the approaches of others.
- Invoke problem-solving strategies, such as illustrating the problem with sketches to clarify the situation or organizing information in a table.
- Explain, when helpful, how to break a problem into simpler parts.
- Solve problems for unknown or undecided quantities by using algebra, graphs, sound reasoning, and other strategies.
- Integrate concepts and techniques from different areas of mathematics.
- Make sensible, reasonable estimates.
- Make justified, logical statements.

Problem Conclusion (Students provide closure to the solution process though summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in grade eight who meet the standard will:

- Verify and interpret results in the context of the original problem.
- Generalize solutions and strategies to address new problem situations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Students' pulse rates vary. What would be considered the normal pulse rate for students in your class? You might want to consider various characteristics or conditions (e.g., exercise) and find out how they relate to pulse rate.
2. In a school of 1,000 lockers, one student opens every locker, another student closes every other locker (second, fourth, sixth, etc.), a third student changes every third locker (opens closed lockers and closes open lockers), and so on, until the thousandth student changes the thousandth locker. At that point, which lockers are open? Why?
3. What is the relationship between a 10-inch pizza and a 14-inch pizza? How much more pizza do you get when you buy a 14-inch pizza than when you buy a 10-inch pizza? If the 10-inch pizza costs $7.75, what would be a fair price for the 14-inch pizza?

Standard 6. Mathematical Communication

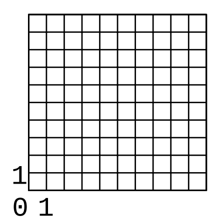
Students communicate their knowledge of basic skills, conceptual understanding, and problem solving and demonstrate their understanding of the mathematical communications of others.

For example, students in grade eight who meet the standard will:

- Use mathematical language and representations with appropriate accuracy; for example, in formulating numerical tables and equations, simple algebraic equations, formulas, charts, graphs, and diagrams.
- Organize work, explain facets of a solution orally and in writing, label drawings, and use other techniques to make meaning clear to the audience.
- Use mathematical language to make complex situations easier to understand.
- Exhibit developing reasoning abilities by justifying statements and defending work.
- Demonstrate understanding of concepts by explaining ideas not only to teachers but also to fellow students or younger children.
- Demonstrate comprehension of mathematical concepts from reading assignments and from other sources.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Imagine that you are working with a team of scientists. You are comparing the movement of ants with that of frogs. One of the scientists says, "If we observe an ant and a frog alone, each for five minutes, the frog will always travel farther." Decide whether you agree with the statement. Draw a diagram that shows the ant's movements for five minutes. Draw a diagram that shows the frog's movements for five minutes. Show all measurements. Use words and pictures to convince the other scientists that your opinion is correct.
2. Imagine that you live on a flat world. The only way to move is through the coordinate plane. You are on a mission to capture a dragon that is threatening your village. If you can find out exactly where it is, you will be able to capture it. Other people in your village have already found out that the dragon does not lie farther north or east than (8, 6). People who have seen the dragon have figured out that it is 4 units long and 1 1/2 units wide. Make up a short story that tells how you looked for the dragon by exploring the coordinate plane. Tell where you think the dragon was found.



3. You have eight cubes. Two of them are painted red; two, white; two, blue; and two, yellow; otherwise they are indistinguishable. You wish to assemble them into one large cube with each color appearing on each face. In how many different ways can you assemble the cube? Draw a diagram of a large cube, showing the color of the faces. Record different solutions. Use words to convey how you can create the large cube.

Grade Eight

This section contains examples of several mathematics tasks with samples of student work for each task. Each task is organized as follows:

- *Task* gives the directions for completing the assignment.
- *Mathematics in the Task* identifies the Challenge Standards addressed in the task (usually more than one) and describes the specific mathematics that students will encounter in completing the task.
- *Setting* describes the situation (in-class, homework, etc.) and the time frame in which the sample work was completed.
- *Samples of Student Work* describes the mathematics evident in each student's work.

Taken together, this collection of tasks is *not* intended to be a test or an assessment of the mathematics standards for students in grade eight. The tasks do not cover the full range of mathematical concepts that students need to know, understand, and be able to apply. They are merely examples of what might be included in an assessment or given as classroom assignments.

Most of the tasks are quite small; generally, they are assignments that may be done in class, as homework, or over a one-week period. Because most of the work is from only a few classrooms and was not intended as a formal assessment, *this document does not identify performance levels for the tasks*. Teachers from different classrooms, schools, and districts need to work together to establish performance levels for formal assessments that will be used to determine whether a student has met the mathematics standards. No single task can provide sufficient information to assess whether a particular student has met a standard. A collection of work from a student will need to be assessed to make a valid judgment.

The samples of student work included with each task are intended to be illustrative of high-quality work that can be done by students in grade eight and may be considered as some evidence that the students are progressing toward meeting the mathematics standards. The work samples are not in each student's handwriting—they were typed into a computer. However, they contain the actual words (and the spelling) and calculations used by the student; and they reflect, as much as possible, the student's drawings, format, and placement of work. Comments about the work describe the mathematical concepts and the approaches the students used as well as any errors they made. Most errors identified in the students' work are minor. The commentaries do not include the way in which the classroom teacher addressed the errors with students. Most tasks include work from different students. The work of a few students is included for a number of tasks. For example, samples of Hope's work (names are fictitious) are shown for several tasks.

The comments about student work in this document are quite detailed; however, teachers are not expected to *write* commentaries similar to these about an individual student's work. Teachers and students are expected to *think* carefully about the mathematics involved in the assignments and tasks they do in their classrooms and the evidence of mathematical knowledge and understanding clearly demonstrated in the student's work. Teachers may want to use the sample tasks and student work shown here as they discuss what the standards mean, how they are reflected in the classroom program, and the way in which they are exemplified in the work students do.

Task: *Can* You Wrap It?

Make a wrapper for a can of food.

1. Experiment with how to make a wrapper for the can using the *minimum* amount of paper.
2. Make the wrapper and draw an exact copy of your wrapper.
3. Calculate the area of your wrapper. Show all your work.
4. Be ready to present your finding to the rest of the class.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
	x			x	x

This task requires students to make a net for a cylinder and to measure and calculate the area and perimeter of circles and rectangles. The task allows students to determine how to approach and solve the problem. It requires students to measure, using appropriate metric units, and to make a model of the cylinder net in the actual size. Students need to realize that the circumference of the lid of the can is the length of the rectangular side and know how to use and record linear and square units.

Setting

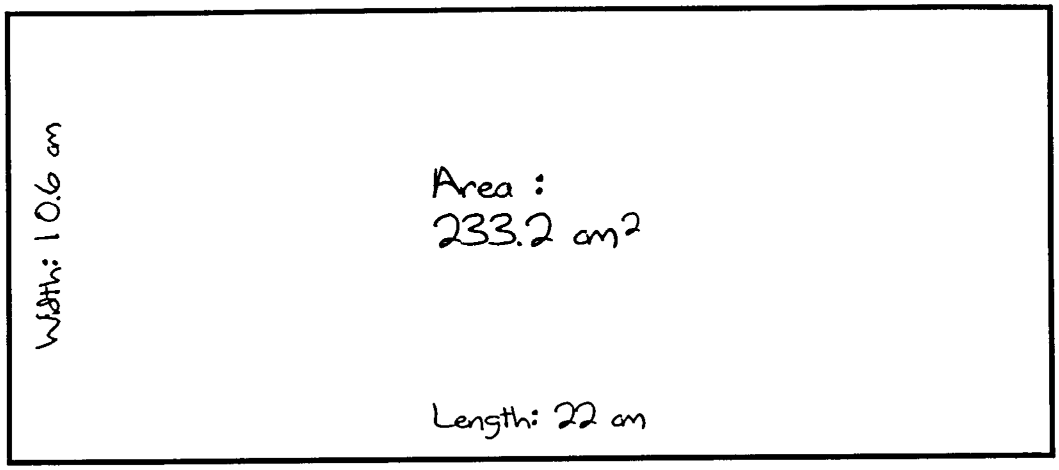
The task was a classroom assignment from a three-week unit on the geometry of circles and was done in the middle of the unit. Students worked with a partner, and the task was completed over two class periods. Each pair of students used a different size can. The first day most students tried different approaches to making a wrapper. The second day students completed their model, calculations, and report, which they turned in. Some pairs presented their work to the class during the second period. Students had tape measures, centimeter rulers, and calculators to use as tools to complete the task.

Samples of Student Work: Hope and Laticia

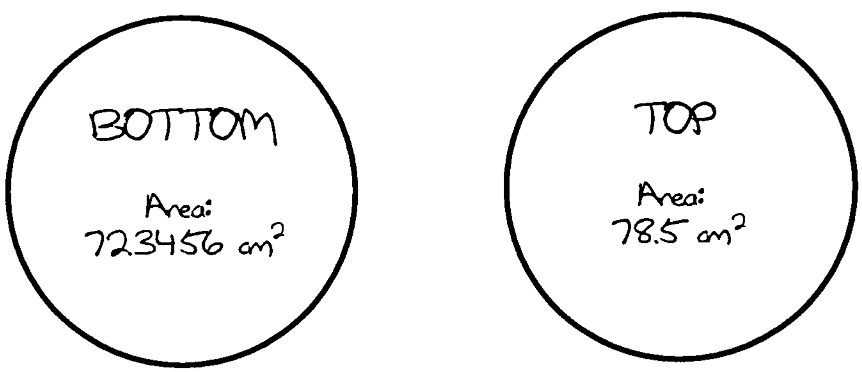
Hope's work shows three unconnected pieces that make the wrapper for her can. The work clearly shows the process used for calculating the areas of the circles and rectangle and the total area of the wrapper. However, the work indicates some lack of understanding of the conventions for writing equations. For example, the equation shown to find the radius of the circle, $d = 10 \div 2 = 5$, may indicate that Hope started with the diameter of 10 and then cut it in half. When questioned by the teacher during class, Hope was adamant that the top and bottom of the can had different diameters: the top measured 10 cm; and the bottom, 9.4 cm. She may have measured inaccurately or she may have varied the way in which she drew around the top and the bottom of the can. Her work includes accurate calculations of two separate circular areas based on her measurements. However, the drawings are mislabeled: the smaller area is written in the larger circle, and the larger area is written in the smaller circle.

Hope's Work (*Can* You Wrap It?)

Hope



Total Area: 78.5 + 723456 + 233.2 =
384.0456 cm²
(Top) (Bottom) (Side)



Top: $d = 10 \div 2 = 5$
$A = 3.14 \times 5 \times 5 = \underline{78.5}$

Bottom: $d = 9.6 \div 2 = 4.8$
$A = 3.14 \times 4.8 \times 4.8 = \underline{72.3456}$

C: (Rectangle side) $L = 22$ $10.6 \times 22 = \underline{233.2}$
$W = 10.6$

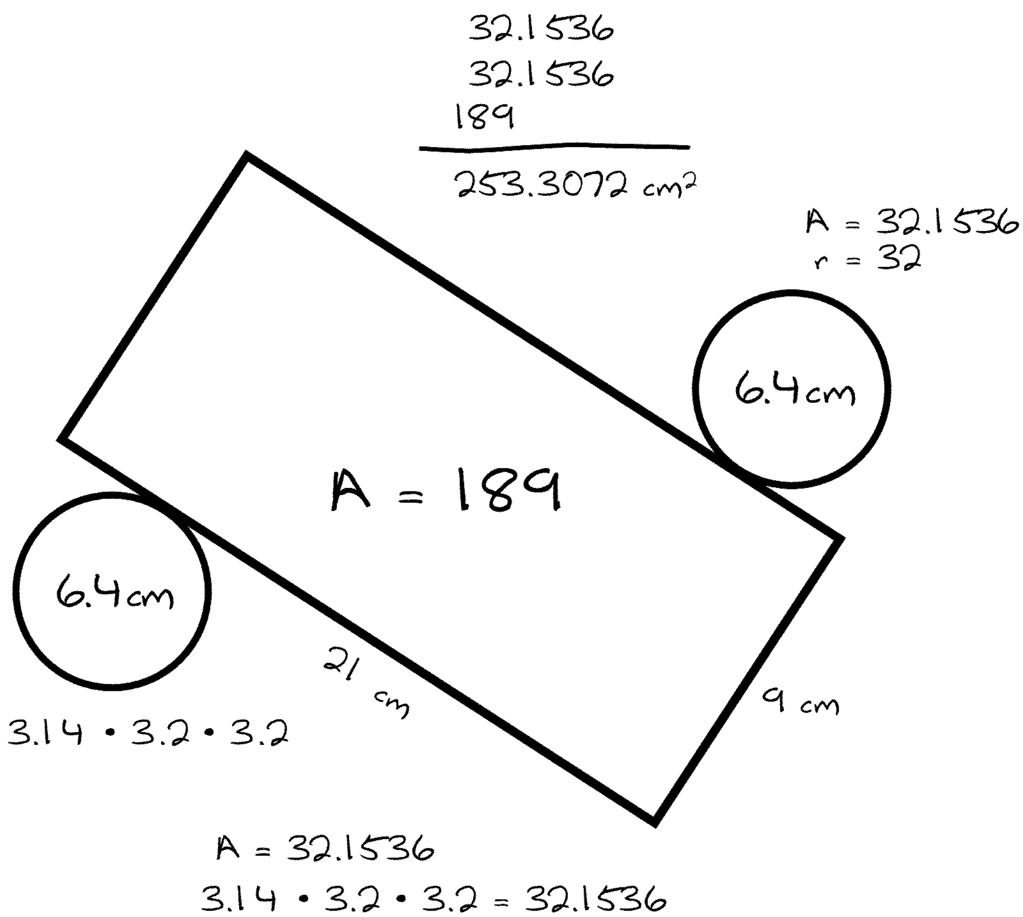
Total Area:
$78.5 + 72.3456 + 233.2 = 384.0456\text{ cm}^2 \approx 384\text{ cm}^2$

Note: Drawings have been reduced for this publication, but the written measurements indicate the actual size of the original drawings.

Laticia's work shows the actual size of the wrapper (two circles attached to a rectangle), the dimensions, and the results of the calculations (the area of each piece and the sum of the areas). The work provides evidence that Laticia was able to find the necessary measures of her can in centimeters and to substitute those values into correct formulas for the areas of circles and rectangles. Her labeling, both of what the measurements represent and of her results, is minimal. For example, the diameters of the circles are indicated only by the measure (6.4 cm) written inside each circle. Above the top circle, however, she correctly indicated that r = 3.2. Although unit labels are not shown for the partial results of the areas of the rectangle and the circle, the correct unit labels of *cm* for the dimensions and cm^2 for square unit are shown for the total area. The calculations are shown in different locations on the paper—the numbers used in calculating the area of each circle are shown at the bottom, and the three partial areas of the wrapper are added at the top.

Laticia's Work (*Can* You Wrap It?)

Laticia



Note: Drawings have been reduced for this publication, but the written measurements indicate the actual size of the original drawings.

Task: Dividing the Dollars

Uncle Sam has $100 and four friends. He gives the first friend 20% of his money. The second friend gets 25% of the money left over; the third friend gets $33^{1}/_{3}$% of the money left after that; and the fourth friend (you) gets 50% of the remaining money. How much money does each person get?

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x				x	x

Students need to understand the situation in which a beginning amount gets reduced by successively larger percentages. They need repeatedly to find a percentage of an amount and to subtract that amount from the balance on hand. They need to show their work in a way that is clear.

Setting

The task was an individual homework assignment completed overnight during an instructional unit on fractions, decimals, and percents.

Samples of Student Work: Hope and Monique

Hope's results are clearly recorded in a chart organized to show the percent and amount that each person received and the amount of money left over after each person received his or her share. The calculations indicate that Hope converted the percent to decimals, calculated the dollar amount to be given, then subtracted that amount from the balance. Converting $33^{1}/_{3}$% to .333 (rather than using the fraction $^{1}/_{3}$) resulted in a discrepancy of two cents. However, in the chart she did an appropriate compensation by rounding $19.98 to $20. The conclusion restates the process she used.

Hope's Work (Dividing the Dollars)

Hope

$ Dividing The Dollars $

Question:

How much money does each of uncle Sam's friends get out of the original $100.00?

Strategies:

~ Chart ~

Person #	% They Get...	$ Given	left over
1	20%	$20.00	$80.00
2	25%	$20.00	$60.00
3	$33\frac{1}{3}$%	$20.00	$40.00
4	50%	$20.00	$20.00

*19.98 ≈ $20.00

They begin at $100

Conclusion:

Uncle Sam's friends each get $20.00. At first you start with $100 then you X it by .20 then you $80.00 and you X that .25 ... and so on...

The big math ideas are dividing, subtracting, percentages, and changing a percentage into a decimal. My work is below.

20% 1st person = $100 X .20 = $20 100
−20

25% 2nd person = $80 X .25 = $20 80
−20

$33\frac{1}{3}$% 3rd person = $60 X .333 = $19.98 ≈ $20 60
−20

50% 4th person = $40 X .50 = $20 40
−20

$20 left

(Probably for Uncle Sam)

Monique’s work shows a sequential, easy-to-follow process. Next to each part of the solution, Monique included a written statement about the amount of money that each friend would receive. The third step of the calculations, rounding $33^{1/3}\%$ to .333, led to a slight miscalculation of the third and fourth persons’ shares. Apparently, she did not realize the problem because the conclusion states that the last friend (Monique) receives more money than the other three do. The final statement, that she receives $1.02 more than the third friend, declares a much larger difference than her own calculations show.

Monique’s Work (Dividing the Dollars)

Monique

Dividing the dollars

1. Problem: How is Uncle Sam going divide his money.

(1)

$$\begin{array}{r} \$100 \\ \times\ .20 \\ \hline \$20.00 \end{array}$$

First friend gets \$20.00 out of \$100 Uncle Sam has.

(2)

$$\begin{array}{r} \$100.00 \\ -\ 20.00 \\ \hline 80.00 \end{array} \qquad \begin{array}{r} 80.00 \\ \times\ .25 \\ \hline \$20.00 \end{array}$$

Second friend gets \$20.00 out of \$80.00 left from the \$80.00 from the first friend

(3)

$$\begin{array}{r} 80.00 \\ -\ 20.00 \\ \hline 60.00 \end{array} \qquad \begin{array}{r} \$60.00 \\ \times\ .333 \\ \hline 19.98 \end{array}$$

Third friend gets 19.98 out of 60.00.

(4)

$$\begin{array}{r} 60.00 \\ -\ 19.98 \\ \hline 40.02 \end{array} \qquad 2\overline{)40.02} = 20.01$$

The fourth friend gets \$20.01 out of 19.98.

Conclusion: The big math idea is %'s and decimals. I think the fourth person gets the most out of the \$1 00. I got 1 penny more than the first and second people and \$1 .02 more than the third.

Task: The Cookie Monster

The cookie monster sneaks into the kitchen and eats one-half of a cookie on the first day. On the second day he comes in and eats one-half of what remains of the cookie from the first day. On the third day he eats one-half of what remains from the second day. If the cookie monster continues this process for seven days, how much of the cookie has he eaten? How much is left? If the process continues, will he ever eat all the cookie? Use pictures, charts, words, or calculations to explain your thinking.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x				x	x

The task requires students to begin with one whole (cookie) and successively take one-half of the remaining part each day for seven days. Students need to organize their data and choose a way in which to demonstrate and verify their conclusions. They need to determine how much of the cookie has been eaten after seven days and how much remains. They also need to discuss whether all the cookie will ever be eaten if the process continues.

Note that the mathematical structure of this task is almost identical to the structure in the task titled “Dividing the Dollars.” The contextual situation is different, and the dollar problem reductions are given in percents rather than in fractions. The cookie problem goes further than the dollar problem does by including an intuitive approach to the mathematical concept of a limit.

Setting

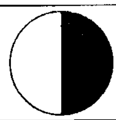
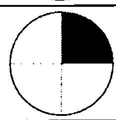
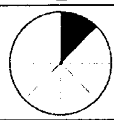
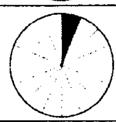
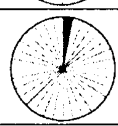
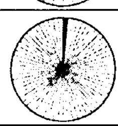
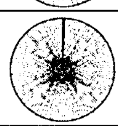
This problem was given during a unit on probability in which students represented parts and wholes by using fractions, decimals, percents, and circle graphs. The problem was introduced on a Monday with a brief group discussion to clarify any questions. Individual work was done at home, and the assignment was turned in on Friday of the same week.

Samples of Student Work: Alejandro and Lucia

Alejandro's work presents the results in a clear, well-organized chart. The first column shows the day; the next three columns show in pictorial, fractional, and decimal or percentage forms the amount of cookie eaten each day; and the last column shows in a fraction the total amount of cookie that has been eaten by the end of each day. The conclusion indicates that Alejandro knows that the fraction 127/128 is expressed in the lowest terms because "127 is prime." He distinguishes the difference between the (mathematical) "mental world" and the "physical world" in the statement that the cookie would never be completely eaten in the first case but "might be finished" in the second case.

Alejandro's Work (The Cookie Monster)

Alejandro

Day	How much of cookie was eatehn	Fraction of cookie eaten	Percentage of cookie eaten	How much of cookie has already been eaten?
1		$\frac{1}{2}$	50%	$\frac{1}{2}$
2		$\frac{1}{4}$	25%	$\frac{3}{4}$
3		$\frac{1}{8}$	.125 ≈ 13%	$\frac{7}{8}$
4		$\frac{1}{16}$	.0625 ≈ 6%	$\frac{15}{16}$
5		$\frac{1}{32}$	.03125 ≈ 3%	$\frac{31}{32}$
6		$\frac{1}{64}$	.015625 ≈ 2%	$\frac{63}{64}$
7		$\frac{1}{128}$	.0078125 ≈ 0.08%	$\frac{127}{128}$

Conclusion: On the seventh day $\frac{127}{128}$ of the Cookie will be eaten since 127 is prime, it's in the lowest terms. $\frac{1}{128}$ of the cookie still remains on the seventh day. If the process continues, he will never finish the cookie, since numbers will go on for etc that cookie will never finish in the mental world, but in the physical world, if you can't divide crums in halves, fourths, eighths, etc., the cookie might be finished.

Lucia's work includes a written narrative describing the process she used, and examples of her work illustrate the description. The work shows how Lucia successively divided a circle in halves and labeled the amounts and the days. The narrative states that she found the size of each subsequent piece by multiplying only the denominators by two because she knew that the numerator of each fraction would be one. The work shows zeros made of dotted lines to indicate that she did not multiply the numerators. This procedure may indicate that she did not realize that multiplying each fractional piece by one-half would be the same as multiplying the denominators by two. The work also includes a chart. The statement under the drawing indicates that she determined 127/128 of the cookie had been eaten by subtracting the remaining 1/128 of the cookie from one, although the wording used is "subtracted 1 to [get] 127/128." The student recognized the dilemma in the question "Will he ever eat all the cookie?" by stating, "I figured out that he will never eat the whole cookie because he can keep eating half until he eats half the atom of the sugar molecule in the piece of cookie."

Lucia's Work (The Cookie Monster)

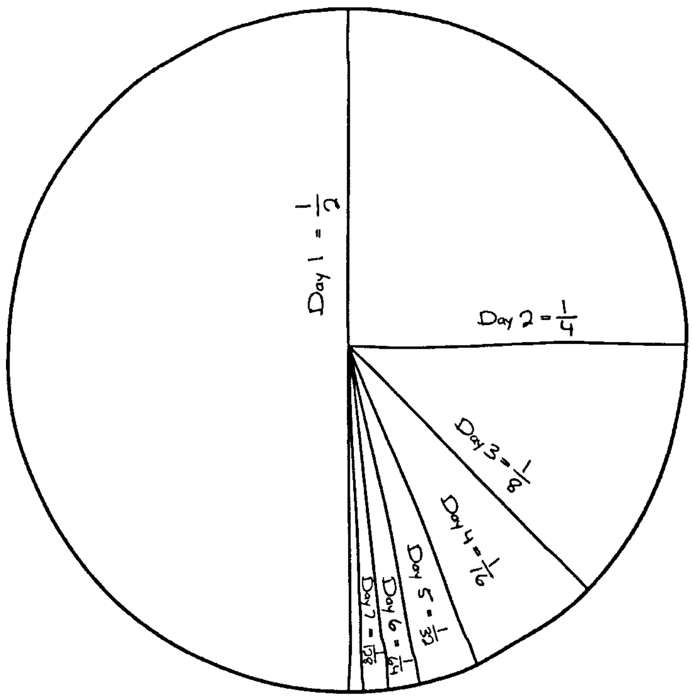
Lucia

Process

I first thought of all my possible ways that I could solve this problem. I choose to start out with a pie graph and work my way from there. I made a pie graph as you can see in the page attached and divided it up seven times as stated in the problem. When I had split the pie graph I started to work on making fractions out of numbers. I knew that on day 1 cookie monster ate of the cookie and on day two he ate half of what was left which came out to be of the cookie. I kept on multiplying the denominator by two and not the numerator because I knew the answer was one of whatever the bottom number came out to be. I came up with the answer that Cookie monster had eaten of the cookie and had of the cookie left to eat. I figured out that he will never eat the whole cookie because he can keep eating half until he eats half the atom of the sugar molecule in the piece of cookie. If you are unclear on how I solved this problem I have examples of my work attached to this paper.

Lucia

Examples of My Work



This is the cookie divided from Day 1 — Day 7

Day 1 $\frac{1}{2} \times \frac{\bigcirc}{2} =$

Day 2 $\frac{1}{4} \times \frac{\bigcirc}{2} =$

Day 3 $\frac{1}{8} \times \frac{\bigcirc}{2} =$

Day 4 $\frac{1}{16} \times \frac{\bigcirc}{2} =$

Day 5 $\frac{1}{32} \times \frac{\bigcirc}{2} =$

Day 6 $\frac{1}{64} \times \frac{\bigcirc}{2} =$

Day 7 $\frac{1}{128}$

To find out how much he has eaten I took $\frac{1}{128}$ which was how much was left and subtracted 1 to $\frac{127}{128}$ for that answer.

List of days and their fractions

Day 1: $\frac{1}{2}$ Day 4: $\frac{1}{16}$ Day 7 = $\frac{1}{128}$

Day 2: $\frac{1}{4}$ Day 5: $\frac{1}{32}$

Day 3: $\frac{1}{28}$ Day 6: $\frac{1}{64}$

1	2	3	4	5	6	7
$\frac{1}{2 \times 2}$	$\frac{1}{4 \times 2}$	$\frac{1}{8 \times 2}$	$\frac{1}{16 \times 2}$	$\frac{1}{32 \times 2}$	$\frac{1}{64 \times 2}$	$\frac{1}{128}$

Times everything by 2

Task: Connecting with a Good Deal

Compare the cost of three long-distance telephone plans on the assumption that you generally make about 14 hours of long-distance phone calls per month in addition to local calls. Which is the best buy? Justify your answer by using T-charts, graphs, or formulas.

Plan 1: Long-distance charges, $0.19 per minute; additional charge of $65 per month for local calls

Plan 2: No charge for local calls; long-distance charges, $0.25 per minute

Plan 3: Unlimited long-distance calling for a flat monthly fee of $249; no charge for local calls

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
	x			x	x

This problem requires students to use numerical data, calculate and compare the results of the costs of three different plans, and determine which is the best buy. Students may represent and compare the three sets of data by using T-charts, equations, and graphs. They need to communicate clearly their process and conclusion.

Setting

The task was a take-home problem. Students had one week in which to complete it. The problem was introduced on a Monday, with students suggesting several strategies they might use to work on it. The assignment was turned in on Friday, and several students shared their solutions with the class. The assignment was part of a unit on an introduction to algebra.

Samples of Student Work: Troy and Kathryn

Troy's work shows the results in the form of T-charts for plans 1 and 2 and shows only the calculated result for plan 3. The chart for plan 1 indicates no cost for zero minutes, an assumption that does not account for the minimum monthly charge of $65. However, the $65 minimum charge is included in all other steps. Both plans 1 and 2 show calculations of charges for the first few minutes, then jump to charges for 840 minutes (or 14 hours) as the typical amount of long-distance time used in a month. The representations of the formula for n minutes show Troy's understanding of functional relationships. Apparently, Troy did not see the confusion caused by using a dot to represent multiplication and another dot to show the decimal point denoting cents. The conclusion is clear, and the calculations are interpreted in the context of the original problem.

Troy's Work (Connecting with a Good Deal)

Troy

Good Deal

Plan 1

Min	$
0	.00
1	$65.19
2	$65.38
3	$65.57
4	$65.76
840	$224.60
n	n•.19+65.00

Plan 2

Min	$
1	.25
2	.50
3	.75
4	$1.00
5	$1.25
840	$210.00
n	n•.25

Plan 3
$249.00

Conclusion: If you make 14 hours a month of long distance calls and you're looking for the chapest plan it'd be plan 2 at .25 per min. For 14 hours. It only comes to $210.00. Plan 3 automatically comes to $249.00. The cheapest plan is two. And Plan 1 is $224.60.

Plan 1 = $224.60
Plan 2 = $210.00 = The best deal
Plan 3 = $249.00

Kathryn's work shows a T-chart for all three plans as well as a graph of the three equations. The work includes a sentence summarizing the cost of each plan. A small error in the sentences about plan 3 states that the plan is "less than plan 3" when it is actually the most expensive plan if you make 14 hours of calls. The summary provides some evidence that Kathryn is beginning to understand that the steepness of the slope and the y-intercept of each equation relate to the solution of the problem and that the points at which the three equations intersect show the break-even points for the three plans. The work goes beyond the assumption of 14 hours of calls by pointing out that the costs of the three plans are similar in the range of 16 to 18 hours of calls.

Kathryn's Work (Connecting with a Good Deal)

Kathryn

plan 1

min	price
0	$65.00
1	65.19
2	65.38
3	65.57
840	$224.60
n	n•19+65

14 hours
x 60 min/hour
840 min.

plan one would average about a $224.60 per month phone bill.
one hour
$76.40

plan 2

min	price
0	$ .00
1	.25
2	.50
3	.75
840	$210.00
n	n.25

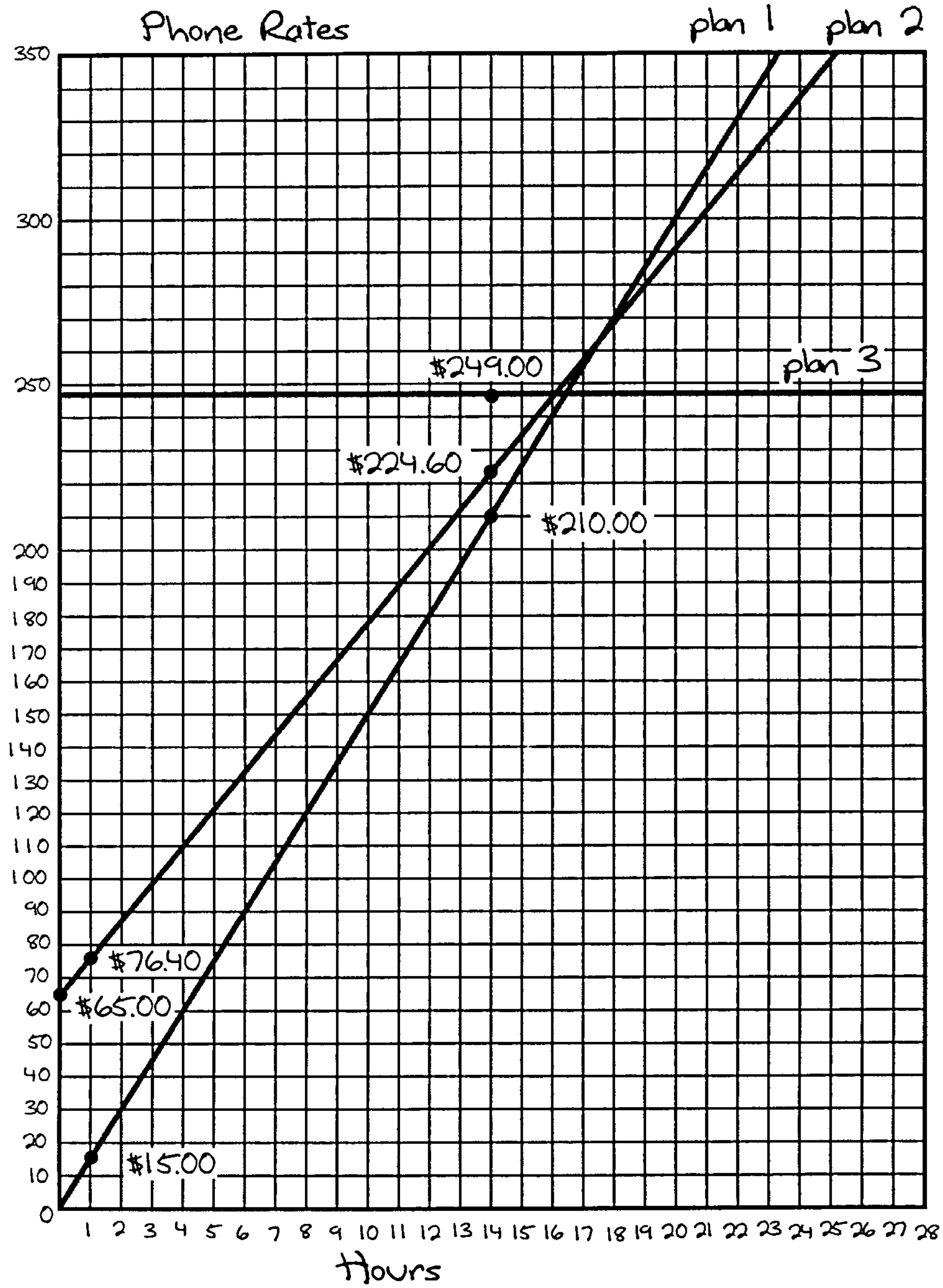
plan two would average about a $210.00 per month phone bill, less then plan 1.
one hour
$15.00
*best

plan 3

min	price
0	$249.00
1	$249.00
2	$249.00
3	$249.00
840	$249.00
n	n=$249.00

plan three will pay $249.00 every month. More then plan 2, less then plan 3.
one hour
$249.00

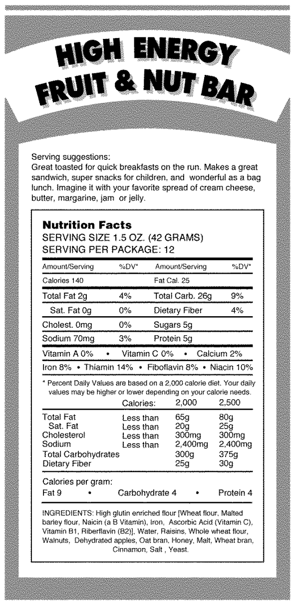
Kathryn



I would choose plan 1 because it's slope is the least steep, well plan 3's slope is really the least steep, but it begins very high. If you spent about 16–18 hours of long distance calling, the prices would be almost exactly the same for 14 and maybe more hours, you should choose plan 1.

Task: Fat Chance Assessment

How healthy is the High Energy Fruit and Nut Bar?



1. List the number of grams of each of the following in the energy bar: fat, carbohydrates, and protein.
2. Calculate the number of calories from each of the following: fat, carbohydrates, and protein.
3. Calculate the total number of calories from the fat, carbohydrates, and protein.
4. Calculate the percent of calories from each: fat, carbohydrates, and protein.
5. Calculate the degrees in a circle graph to show the percent of calories from the fat, carbohydrates, and protein.
6. Using a protractor and a compass, draw a circle graph showing the percent of fat, carbohydrates, and protein in the energy bar.
7. Compare the energy-bar circle graph with the recommended diet. Is an energy bar a healthy snack? Why or why not? Explain.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
	x			x	x

The items in this task require students to read and interpret a food label related to amounts of fat, carbohydrates, and protein in the food. Students need to convert the grams of each food element into calories and find the percent of the calories for each element compared with the total number of calories in the food. Students are asked to represent these percentages in a circle graph and show their calculations for the angles of the circle graph. Finally, students must compare their results with a recommended standard for fat content and determine whether the fruit-and-nut bar is a healthy snack.

Setting

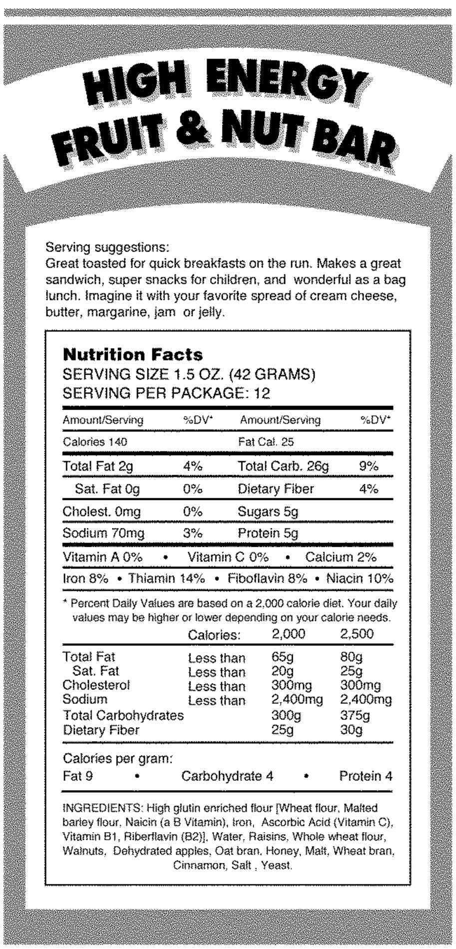
The questions were part of the assessment given at the end of a unit on fractions decimals, and percents. The full assessment, which included a number of other items, was completed within a 45-minute class period.

Samples of Student Work: Hope

Hope's work provides evidence that Hope is able to gather the necessary data from the food label. The work shows accurate calculations in converting grams to calories; finding the total number of calories; and determining the percentages of fat, carbohydrate, and fat calories in the energy bar. The work shows correct calculations of degrees for the circle graph, with appropriate rounding of results. The circle graph is accurately drawn. The conclusion (item 7) states that the energy bar is healthy because the fat content is less than 30%, which is the recommended standard for fat in the diet.

Hope's Work: Fat Chance Assessment

Fat Chance Assessment Hope



How healthy is the High Energy Fruit and Nut Bar?

1. List the number of grams of each of the following in the energy bar: fat, carbohydrates, and protein.

   Fat = 2 g   Protein = 5 g
   Carbo. = 26 g

2. Calculate the number of calories from each of the following: fat, carbohydrates, and protein.

   protein carbo
   9 cal/g 5 g 26 g
   2g Fat 5 cal/g x 4 cal/g
   18 CAL 20 CAL 104 CAL

3. Calculate the total number of calories from the fat, carbohydrates, and protein.

   18 cal + 20 + 104 = 142 (Total) Calories

4. Calculate the percent of calories from each: fat, carbohydrates, and protein.

   $\frac{13}{142} = 13\%$ $\frac{20}{142} = 14\%$ $\frac{102}{142} = 73\%$
   Fat   Proteinc   Carbo

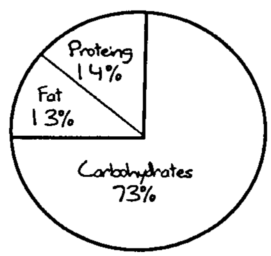
5. Calculate the degrees in a circle graph to show the percent of calories from the fat, carbohydrates, and protein.

Fat	Pro	Carbo
13% = .13	14% = .14	73% = .73
13 x 360°=	.14 x 360°=	.73 x 360°=
rounded off → 46.8° ≈ 47°	50.4° ≈ 50°	262.8° ≈ 263°

47° + 50° + 263° = 360°

6. Using a protractor and a compass, draw a circle graph showing the percent of fat, carbohydrates, and protein in the energy bar.

fat = 47°
protein = 50°
carbohydrates = 263°



* and show what percentage they of something

Example:
$\frac{25}{100}$ = 0.25 = 25%

7. Compare the energy-bar circle graph with the recommended diet. Is an energy bar a healthy snack? Why or why not? Explain.

Yes, the Energy Bar is healthy. Except for the carbohydrates and a little protien going over, it is healthy. Because the Fat is 13% and is under the recommended diet of 30% Fat.

Task: The Pizza Problem

What is the relationship between a 10-inch pizza and a 14-inch pizza? How much more pizza do you get when you buy a 14-inch pizza than when you buy a 10-inch pizza? If the 10-inch pizza costs $7.75, what would be a fair price for the 14-inch pizza? Why?

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
	x			x	x

The task requires students to connect their understanding of circles and areas to a real-world decision of fair pricing. Students need to recognize that the data supplied refer to the diameters of the pizzas but, to determine a fair price, they need to calculate and compare the *areas* of the circles. Finally, they need to determine the ratio of the areas of the two circles and apply the same ratio to the prices. (Some students may set up a proportion to find the price, but most students simply double the price because the area of the 14-inch pizza is about twice the area of the 10-inch pizza.)

Setting

This assignment was an open-ended item in an assessment of a unit on area and perimeter. Students worked individually, completing the assessment in one class period.

Samples of Student Work: Elaine and Tonya

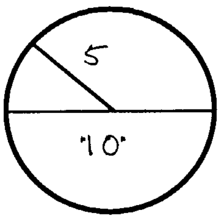
Elaine's work notes the differences between the diameters of the two pizzas, then gives the correct area of each pizza. Sketches of the pizzas and the calculations beside the narrative show that Elaine used the correct measurements to find the areas. However, the work exhibits an incorrect use of equation notation; that is, $10 \div 2 = 5 \text{ x } 5 = 25 \text{ x } 3.14 = 78.5$. This string of calculations appears to be a record of the process Elaine used. The work also shows that she subtracted the area of the smaller pizza from the area of the larger one and used the difference of 75.36 square inches as the reason for stating that the larger pizza "has *almost* twice as much area." The work shows that Elaine determined the cost for the 14-inch pizza by first finding the price per square inch of the 10-inch pizza (.099 or about 10¢). Then she calculated the cost of the additional 75.16 square inches of the 14-inch pizza ($7.536) and added that sum to the cost of the 10-inch pizza. The conclusion includes a statement that restaurants often charge less for larger amounts.

Tonya's work shows sketches of two pizzas (the sketch of the 10-inch pizza is larger than that of the 14-inch pizza) and lists of the radius, diameter, circumference, and area (without the unit labels) of each circle. The work does not show the calculations used to determine the radii, circumferences, or areas of the circles; yet all are correct. There is an incorrect unit label of inches instead of square inches in the statement that "you will get 75.36 inches more." The work indicates that Tonya subtracted the two areas to find the difference between the two, concluding that "the 10 inch pizza is about half of the 14 inch pizza," and doubled the $7.75 price to get a price of $15.50 for the 14-inch pizza.

Elaine's Work (The Pizza Problem)

Elaine

The Pizza Problem

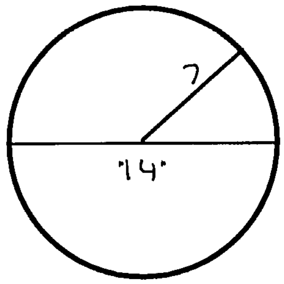


Area: 78.5

10÷2 = 5x25x3.14=78.5

  153.86
- 78.50
  75.36

78.5÷7.75=.099 ≈ $.10



Area: 153.86

7x7=49x3.14=153.86

  75.36
x .10
  7.536

The relationship between a 10 and fourteen inch pizza is 4 inches in diameter.

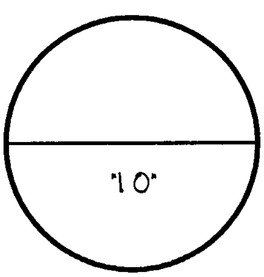
The 14" pizza has an area of 153.86. The 10" pizza has an area of 78.5. The 14" has <u>almost</u> twice as much area. This means that the 14" is almost twice as large. You get 75.36 in² more when you get a 14" pizza. A good price for a 14" pizza would be $15.29. This is because each square inch costs about 10¢.

  7.536
  7.750
  15.286

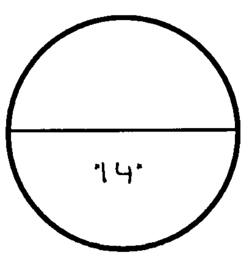
The 14" pizza has 75.36 in² more than the 10". The 14" pizza has more than the 10" That would be $7.536 more. If you add that to 7.75 you get about 15.29. That would be a fair price for a fourteen inch pizza. Usually the places charge less than that because you are buying more, so their prices would probably be less.

Tonya

Pizza Problem



radius = 5
diameter = 10
circumference 31.4
area = 78.5



radius = 7
diameter = 14
circumference = 43.96
area = 153.86

If you buy a 14 inch pizza instead of a 10 inch pizza you will get 75.36 inches more. I got this answer by subtracting the 10 inch area from the 14 inch area. Since the 10 inch pizza is about half of the 14 inch pizza the 14 inch pizza should be almost twice the amount of the 10 inch pizza, so I think the price should be $15.50.

First-Year Algebra

Standard 1. Number and Operations

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in number and operations.

For example, students in algebra I who meet the standard will:

- Demonstrate how and when to use the operations of arithmetic, the properties of those operations, and basic powers and roots.
- Write mathematical expressions and sentences that satisfy given conditions using appropriate numerical and algebraic symbols.
- Evaluate formulas and other expressions using the rules for the order of operations.
- Locate rational and irrational numbers on the number line and graph points on the coordinate plane.
- Demonstrate facility in the use of numbers in the form of ratios, proportions, and percents.
- Use appropriate computational techniques and tools, supported by estimation strategies, to solve real-life and mathematical problems.
- Find permutations (arrangements) and combinations (selections) in real-life and mathematical situations using concrete objects and tree diagrams.
- Compare numbers of different magnitude using order relations, differences, ratios, proportions, percents, and proportional change.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Given that x is a perfect square, find a formula for the next integer greater than x that is a perfect square.
2. Investigate whether or not one can find four odd numbers whose sum is 27. Explain your reasoning and justify your conclusion. If possible, generalize your conclusion.

3. Show that the product of the least common multiple of two different integers and the greatest common divisor of the same two integers are the same as the product of the two integers.
4. Present a convincing argument for the need to have irrational numbers.
5. There are x^{12} bacteria in a given mold at 10:10 a.m. Each bacterium produces offspring at the rate of 2^5 per minute. Assuming that none had died, how many bacteria exist at 10:11 a.m.?
6. Find ten right triangles that satisfy these two conditions: (A) all three sides are integers with no common factors; (B) the length of one of the legs is an even integer, and the length of the other is an odd integer. For each of the ten triangles, find the difference between the hypotenuse and the side that is an even integer. Do you see a pattern? Show algebraically that your conjecture is always true.
7. Find the greatest and the least values of a certain number that when rounded to one decimal place is 2.6 and when rounded to two decimal places is 2.65, and locate the point associated with this number on the number line.

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in algebra I who meet the standard will:

- Connect the visual and algebraic representations of lines with their geometric characteristics, such as slope, intercepts, parallelism, and perpendicularity.
- Use scientific notation to represent large and small measurements and solve problems using this notation.
- Use coordinate geometry to find midpoints of line segments and determine areas of standard geometric figures.
- Select appropriate units of measure and apply constraints of significant digits when computing with them so as to achieve the precision and accuracy of measurement desired.
- Use measurements in a variety of interdisciplinary applications that include dimensional analysis.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards are listed as follows:

1. A pipe one inch in diameter is delivering two gallons of water per minute. What is the rate of flow in cubic inches per second? (There are 231 cubic inches per gallon.)
2. Three different measuring instruments are used to find that the three sides of a triangle are 5.7 centimeters, 12.345 centimeters, and 14.23 centimeters. What is the most accurate approximation of the perimeter of the triangle?
3. Find the measures of the angles of a triangle if the angle measures are in a 1:2:3 ratio.

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in algebra I who meet the standard will:

- Use techniques of symbolic manipulations.
- Solve linear, quadratic, and exponential equations using experimentation, symbolic manipulations, and/or graphical techniques.
- Use functions to represent mathematical patterns.
- Use right-triangle trigonometry to solve real-life applications (indirect measurements of height and distance).
- Use multiple representations of quantitative relationships—verbal, graphic, algebraic, numerical, or diagrammatic (e.g., rate relationships).
- Define and use variables, parameters, and constants in work with both functions and equations.
- Model given situations and interpret given functions in terms of situations.
- Represent functional relationships in formulas, tables, and graphs and translate among those representations.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. The length of a rectangle of fixed area equals that area divided by its width. What happens to the width of the rectangle if its length is halved?

2. If an integer, x, is a perfect square of n ($x = n^2$), find a formula for the least integer after x that is a perfect square. Explain and justify your reasoning.
3. Determine whether the product of four consecutive positive integers can be a perfect square. Justify your conclusion.

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in algebra I who meet the standard will:

- Collect, organize, display, and analyze single-variable data using frequency distributions, histograms, and summary statistics.
- Use sampling techniques to make predictions about large populations.
- Use simple counting procedures (tree diagrams, multiplication principle, and so forth) to show all possible arrangements or combinations.
- Use experimental or theoretical probability or both to represent and solve problems.
- Use the mathematical ideas of dispersion of data (range, standard deviation of the mean, box and whisker plots, or stem and leaf plots) to summarize and communicate information about collections.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Over the past five years, it rained 18, 12, 15, 14, and 20 days during the year. Approximately how many days would you expect it to rain this year?
2. When any two integers from 1 through 9 are selected at random and then added, determine the possible sums and the probability of each; generalize to two integers 1 through n.
3. Contributions to a political campaign were $15,000, $15,000, $15,000, $18,000, $18,000, $25,000, and $90,000. The candidate said that the average contribution was $15,000; the newspaper said the average contribution was $18,000; and the opposition party said the average was $28,000. Discuss and interpret the meaning of the three statements.

4. You hear that your new neighbor has two children. You see one child, a girl, playing in the backyard. What is the probability that both children are girls? Using concrete materials, design an experiment that you could use to simulate this situation.
5. Count the number of raisins in several small individual boxes of cereal. Display the resulting data in at least three different ways. Indicate which representation of the data best shows an overview of the data.

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in algebra I who meet the standard will:

- Fill out the formulation of a definite problem to be solved.
- Extract pertinent information from the situation as a basis for working on the problem.
- Ask and answer a series of appropriate questions to find a solution and do so with minimum “scaffolding” in the form of detailed guiding questions.
- Identify mathematical problems whose solution leads to a useful resolution of given situations.
- State questions with answers that could be derived from the information given in a situation.
- Identify mathematical questions that arise from situations or models of situations (e.g., features of the work environment, manipulatives, sketches, graphs, tables, diagrams, and so forth).

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in algebra I who meet the standard will:

- Formulate reasonable mathematical hypotheses or conjectures based on a general description of a situation and identify any missing or extraneous information that might help or hinder the resolution of the situation.
- Select a variety of strategies or approaches to solve problems.

- Make connections among mathematical concepts or skills in the solution of problems.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in algebra I who meet the standard will:

- Verify the accuracy and validity of the mathematical procedures used to solve problems.
- Express problem-solving strategies as a general rule and extend strategies to other situations.
- Formulate generalizations of the results obtained.
- Carry out extensions of the given problem to related problems.

Mathematical Reasoning. (Students not only make observations and state results but also justify or prove why the results hold in general.)

For example, students in algebra I who meet the standard will:

- Show forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including using deductive and inductive reasoning, making and testing conjectures, and using counterexamples and indirect proof.
- Differentiate clearly between giving examples that support a conjecture and giving a proof of the conjecture.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Make a study of student absences caused by illness and find predictors for students in the top 10 percent and the lowest 10 percent of frequency of absences. Look for predictors related to diet, amount of sleep, ratio of weight to height, and so forth.
2. Suppose you work for an unstable company and your salary undergoes changes over a three-month period. In each of the following situations, discuss how your salary at the end of the three-month period compares with your salary at the beginning of that period.
   a. Your salary is cut 10 percent and three months later is raised 10 percent.
   b. Your salary is cut 20 percent and three months later is raised 25 percent.
   c. Your salary is raised 10 percent and three months later is cut 10 percent.

3. Develop a convincing argument to explain why the product of two negative numbers is a positive number.
4. Explain how you could decide whether the following game is fair or unfair:

   This game is played with two ordinary dice of different colors (e.g., one white and one red). Both dice are thrown at the same time. The player wins if the number on the white die is greater than the number on the red die. The player loses in all other cases.

5. A politician stated in a speech that an increase in a budget item from $10,000,000 to $40,000,000 is a 300 percent increase; therefore, a decrease from $40,000,000 to $10,000,000 is a decrease of 300 percent. Explain whether or not this reasoning is correct.
6. In an elementary school district, it was decided to spend at least 8,800 minutes of instructional time per school year on mathematics in each class. What questions would have to be asked and answered in order to develop an instructional plan to meet that requirement?

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, understanding of concepts, and ability to solve problems and understand mathematical communications of others.

For example, students in algebra I who meet the standard will:

- Use basic mathematical vocabulary and terminology, standard notation and symbols, common conventions for graphing, and general features of effective mathematical communication styles.
- Use mathematical representations with appropriate accuracy, including numerical tables, formulas, functions, algebraic equations, charts, graphs, and diagrams.
- Interpret and explain the problem.
- Present mathematical procedures and results clearly, systematically, succinctly, and correctly.
- Describe and discuss mathematical ideas effectively both orally and in writing.
- Explain mathematical concepts or ideas clearly to peers or others.
- Read with understanding mathematical text and other writing about mathematics.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Write a computer program to solve equations of the form $Ax + By = C$, in which A, B, and C are any three real numbers. Explain what the relationship between A, B, and C must be for the solution to be an integer.
2. Explain, using words and graphs, what effect adding, subtracting, or multiplying x^2 by a number has on the graph of $y = x^2$.
3. Write an explanation for a sixth grade mathematics class showing why you get the results that occur in the following investigation:

   Pick your favorite number from one through nine. Multiply it by nine. Multiply that answer by 12,345,679. How is your result related to your favorite number?

4. Make and defend a conjecture about the evenness or oddness of the product of two or more consecutive integers.

Geometry

Standard 1. **Number And Operations**

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in number and operations.

For example, students in geometry who meet the standard will:

- Demonstrate understanding of the concept of the distance between two points and their difference on a number line.
- Use the one-to-one correspondence between the set of ordered pairs of real numbers (x,y) and the set of points in a plane in the development of various geometric entities.
- Compare numbers of different magnitude using order relations, differences, ratios, proportions, percents, and proportional change.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Show that between any two rational numbers there is always a third rational number. Discuss what your conclusion implies about the number of points between any two points on a line.
2. Show that the distances from the midpoint of the hypotenuse of any right triangle to any vertex of the triangle are equal.
3. Find the distance between home plate and second base on a baseball diamond if the distance between each base is 90 ft. and a baseball diamond is a square.

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in geometry who meet the standard will:

- Use geometric measures of length, area, surface area, volume, and angle; and use nongeometric measures of weight, monetary value, and time.
- Use inductive and deductive techniques, including graphing technology, to develop properties of geometric shapes and relationships between them (such as similarity and congruence).
- Use synthetic, coordinate, and/or transformational geometry in direct or indirect proof of geometric relationships.
- Use standard measurement systems, both metric and U.S. customary.
- Select appropriate units of measures to achieve the precision and accuracy desired for a given situation.
- Use right-triangle geometry, similarity, and trigonometric ratios in the solution of real-life applications (Pythagorean theorem or indirect measurements of height and distance).

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. A plane geometric figure has a perimeter of 120. What is the area if the figure is an equilateral triangle? A square? A regular hexagon? A regular octagon? What can be said about the relative areas of two regular polygons having 12 sides and 15 sides, respectively. Generalize for n sides. What is the shape of the regular figure that will have the greatest area possible and a perimeter of 120?
2. Decide which way to roll up an $8^1/_2$- by 11-inch piece of paper to get a piece of a cylindrical surface that has the greatest surface area. If you had a top and bottom, which cylinder would have the greatest volume? Justify your answer.
3. Given a 3, 4, 5 right triangle, find the measure of the smallest angle of the triangle to the nearest degree.
4. For a regular polygon of m sides that rolls around a stationary regular polygon of n sides with the same side length, how many times does the first polygon rotate about the second, stationary polygon? How many times does it make a complete revolution before the starting position on both polygons is reached again?
5. Design a scale model of the solar system. Does your scale model fit into your classroom? Your campus? Explain why or why not.

Standard 3. Function and Algebra

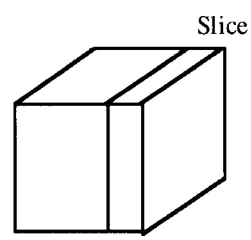
Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in geometry who meet the standard will:

- Use algebraic concepts and skills in applications (e.g., problems involving the Pythagorean theorem and volume).
- Solve problems involving proportional reasoning and similarity relationships.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

> If a one-inch slice taken from a wooden cube left a rectangular solid with a volume of 48 cubic inches, what was the edge length of the original cube?



Standard 4. Statistics and Probability

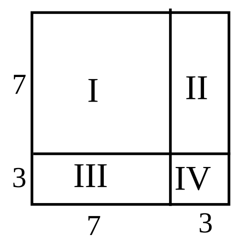
Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in geometry who meet the standard will:

- Use and interpret problems involving probability in terms of areas of geometric figures.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

> Which of the areas in this diagram of a square region could represent the probability that a basketball player who has a 70 percent probability of making a basket will make two consecutive baskets?



Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in geometry who meet the standard will:

- Fill out the formulation of a definite problem to be solved.
- Extract pertinent information from the situation as a basis for working on the problem.
- Ask and answer a series of appropriate questions to find a solution and do so with minimum “scaffolding” in the form of detailed guiding questions.
- Identify mathematical problems whose solution leads to a useful resolution of given situations.
- State questions with answers that could be derived from the information given in a situation.
- Identify mathematical questions that arise from situations or models of situations (e.g., features of the work environment, manipulatives, sketches, graphs, tables, diagrams, and so forth).

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in geometry who meet the standard will:

- Formulate reasonable mathematical hypotheses or conjectures based on a general description of a situation and identify any missing or extraneous information that might help or hinder the resolution of the situation.
- Select a variety of strategies or approaches to solve problems.
- Make connections among mathematical concepts or skills in the solution of problems.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in geometry who meet the standard will:

- Verify the accuracy and validity of the mathematical procedures used to solve problems.
- Express problem-solving strategies as a general rule and extend strategies to other situations.
- Formulate generalizations of the results obtained.
- Carry out extensions of the given problem to related problems.

Mathematical Reasoning (Students not only make observations and state results but also justify or prove why the results hold in general.)

For example, students in geometry who meet the standard will:

- Show forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures, and using counterexamples and indirect proof.
- Differentiate clearly between giving examples that support a conjecture and giving a proof of the conjecture.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Find the dimensions of three right triangles such that in each triangle the lengths of the three sides and the length of the altitude to the hypotenuse are all integers.
2. Describe at least two situations in which the result of a calculation involving measurement must be rounded up (e.g., 8.4 inches rounded up to 9 inches) and at least two situations in which the result of a calculation must be rounded down (e.g., 8 $^3/_4$ inches rounded down to 8 inches).

3. Design a running track with two semicircular ends and two parallel straightaways. If the track is to be exactly 440 yards around, explain how you would decide the length of the straightaway sections and the diameter of the semicircles.

Standard 6. Mathematical Communication

Students communicate their knowledge of basic skills, understanding of concepts, and ability to solve problems and understand mathematical communications of others.

For example, students in geometry who meet the standard will:

- Use basic mathematical vocabulary and terminology, standard notation and symbols, common conventions for graphing, and general features of effective mathematical communication styles.
- Use mathematical representations with appropriate accuracy, including numerical tables, formulas, functions, algebraic equations, charts, graphs, and diagrams.
- Interpret and explain the problem.
- Present mathematical procedures and results clearly, systematically, succinctly, and correctly.
- Describe and discuss mathematical ideas effectively both orally and in writing.
- Explain mathematical concepts or ideas clearly to peers or others.
- Read with understanding mathematical text and other writing about mathematics.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. When a newspaper ad was redesigned, the original column length was increased by one-half. To keep the area of the new ad the same as the area of the original ad, the original width was reduced by two-thirds. Explain whether or not this is correct and justify your reasoning.
2. Given the volume (C) and the floor area (F) of the passenger cabin in a commercial aircraft, what is the theoretical maximum of the number of seats that can be placed in the cabin if each passenger and crew member has no less than 5.5 cubic feet of free space and a seat-print (on the cabin floor) of between 4.7 and 5.8 square feet? The aisle space needs to be at least 104 feet long and 2.5 feet wide in each of the two aisles. What decision factors will further limit the number of seats that can be placed in the cabin?

High School (Integrated)

Standard 1. Number and Operations

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in number and operations.

For example, students in high school who meet the standard will:

- Demonstrate how and when to use the operations of arithmetic, the properties of those operations, and basic powers and roots.
- Write mathematical expressions and sentences that satisfy given conditions using appropriate numerical and algebraic symbols.
- Evaluate formulas and other expressions using the rules for the order of operations.
- Locate rational and irrational numbers on the number line and graph points on the coordinate plane.
- Demonstrate facility in the use of numbers in the form of ratios, proportions, and percents.
- Use appropriate computational techniques and tools, supported by estimation strategies, to solve real-life and mathematical problems.
- Find permutations (arrangements) and combinations (selections) in real-life and mathematical situations using concrete objects and tree diagrams.
- Compare numbers of different magnitude using order relations, differences, ratios, proportions, percents, and proportional change.
- Demonstrate understanding of the concept of the distance between two points and their difference on a number line.
- Use the one-to-one correspondence between the set of ordered pairs of real numbers (x,y) and the set of points in a plane in the development of various geometric entities.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Figure out the smallest and largest values of a certain number which rounded to one decimal place is 2.6 and rounded to two decimal places is 2.65 and illustrate it on a number line.
2. Show that there must have been at least one misprint in a newspaper report on an election which read:

   Yes votes 13,657 (42 percent)
   No votes 186,491 (58 percent)

   and suggest two different specific places where a misprint might have occurred.
3. Given an infinite four-column table with first row 1, 2, 3, 4; second row 5, 6, 7, 8; and so forth, show that if any number from the second column is added to any number from the third column, the result will be in the first column. Generalize to other combinations of columns and generalize to a seven-column table.
4. If 10 percent of U.S. citizens have a certain trait and four out of five with that trait are men, what proportion of men have that trait? What proportion of women have it? Explain whether the answer depends on the proportion of U.S. citizens who are women, and if so, how.
5. Show that the least common multiple of a and b, written $[a, b]$, and the greatest common divisor of a and b, written (a, b), are related by $[a, b]\,(a, b) = ab$.
6. Show how to enlarge a picture by a factor of 2 using repeated enlargements at a fixed setting on a photocopy machine that can enlarge only up to 155 percent. Do the same for enlargements by a factor of 3, 4, and 5.
7. Calibrate and check a bicycle odometer. Using data collected from an incorrectly set odometer, show how to take data about a trip; then show how to convert those figures into accurate data.
8. Create a schedule for a ping-pong tournament among 10 players in which each one plays every other player exactly once, arranging the schedule so that no players have to sit out while the others are playing. Try to do the same for a tournament with 16 players. Then (this is much harder) say what you can about the general case of a tournament with $2n$ players. Create effective and revealing representations for the schedules.

9. Show that the product of the least common multiple of two different integers and the greatest common divisor of the same two integers are the same as the product of the two integers.
10. Present a convincing argument for the need to have irrational numbers.

Standard 2. Geometry and Measurement

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in geometry and measurement.

For example, students in high school who meet the standard will:

- Connect the visual and algebraic representations of lines with their geometric characteristics, such as slope, intercepts, parallelism, and perpendicularity.
- Use scientific notation to represent large and small measurements and solve problems using this notation.
- Use coordinate geometry to find midpoints of line segments and determine areas of standard geometric figures.
- Select appropriate units of measure and apply constraints of significant digits when computing with them so as to achieve the precision and accuracy of measurement desired.
- Use measurements in a variety of interdisciplinary applications that include dimensional analysis.
- Use geometric measures of length, area, surface area, volume, and angle; and use nongeometric measures of weight, monetary value, and time.
- Use inductive and deductive techniques, including graphing technology, to develop properties of geometric shapes and relationships between them (such as similarity and congruence).
- Use synthetic, coordinate, and/or transformational geometry in direct or indirect proof of geometric relationships.
- Analyze geometric patterns, including sequences of growing shapes, and characterize the patterns in terms of properties of the n^{th} stage.
- Use standard measurement systems, both metric and U.S. customary.
- Select appropriate units of measure to achieve the precision and accuracy desired for a given situation.
- Use right-triangle geometry, similarity, and trigonometric ratios in the solution of real-life applications (Pythagorean theorem or indirect measurements of height and distance).

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Use *pi* (π) appropriately when calculating the area and circumference of a circle.
2. Using a scale and a compass, estimate the difference between two points on a map.
3. Estimate the time required to travel between two points on a map by using an average speed.
4. A plane geometric figure has a perimeter of 120. What is the area if the figure is an equilateral triangle? A square? A regular hexagon? A regular octagon? What can be said about the relative areas of two regular polygons having 12 sides and 15 sides, respectively. Generalize for *n* sides. What is the shape of the regular figure that will have the greatest area possible and have a perimeter of 120?
5. Decide which way to roll up an $8^1/_2$- by 11-inch piece of paper to get a piece of a cylindrical surface that has the greatest surface area. If you had a top and bottom, which cylinder would have the greatest volume? Justify your answer.
6. For a regular polygon of *m* sides that rolls around a stationary regular polygon of *n* sides with the same side length, how many times does the first polygon rotate about the second, stationary polygon? How many times does it make a complete revolution before the starting position on both polygons is reached again?
7. Design a scale model of the solar system. Does your scale model fit into your classroom? Your campus? Explain why or why not.

Standard 3. Function and Algebra

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in function and algebra.

For example, students in high school who meet the standard will:

- Use techniques of symbolic manipulations.
- Solve linear, quadratic, and exponential equations using experimentation, symbolic manipulations, and/or graphical techniques.
- Use functions to represent mathematical patterns.
- Use right-triangle trigonometry to solve real-life applications (indirect measurements of height and distance).
- Use multiple representations of quantitative relationships—verbal, graphic, algebraic, numerical, or diagrammatic (e.g., rate relationships).
- Define and use variables, parameters, and constants in work with both functions and equations.
- Model given situations and interpret given functions in terms of situations.
- Represent functional relationships in formulas, tables, and graphs and translate among those representations.
- Use algebraic concepts and skills in applications (e.g., problems involving the Pythagorean theorem and volume).
- Solve problems involving proportional reasoning and similarity relationships.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. A certain bacterium divides every 20 minutes. This table gives the total number of bacteria after several 20-minute time intervals. Describe a method of predicting the number of bacteria that will appear after one hundred 20-minute intervals.

Minutes:	0	20	40	60
Number:	1,000	2,000	4,000	8,000

2. If an integer x is a perfect square, find a formula for the smallest integer after x that is a perfect square. Explain and justify your reasoning.
3. Determine whether the product of four consecutive positive integers can be a perfect square. Justify your conclusion.

Standard 4. Statistics and Probability

Students demonstrate their knowledge of basic skills, conceptual understanding, and problem solving in statistics and probability.

For example, students in high school who meet the standard will:

- Collect, organize, display, and analyze single-variable data using frequency distributions, histograms, and summary statistics.
- Use sampling techniques to make predictions about large populations.
- Use simple counting procedures (tree diagrams, multiplication principle, and so forth) to show all possible arrangements or combinations.
- Use experimental or theoretical probability or both to represent and solve problems.
- Use the mathematical ideas of dispersion of data (range, standard deviation of the mean, box and whisker plots, or stem and leaf plots) to summarize and communicate information about collections.
- Use and interpret problems involving probability in terms of areas of geometric figures.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Contributions to a political campaign were $15,000, $15,000, $15,000, $18,000, $18,000, $25,000, and $90,000. The candidate said that the average contribution was $15,000; the newspaper said the average contribution was $18,000; and the opposition party said the average was $28,000. Discuss and interpret the meaning of the three statements.
2. The results of tossing a die 100 times were:

Outcome:	1	2	3	4	5	6
Number of times:	10	12	12	33	22	11

   Discuss the likelihood that the die is “loaded.” Use mathematical arguments to support your position.

3. When any two integers between 1 and 9 are selected at random and then added, determine the possible sums and the probability of each; generalize to two integers 1 through *n*.

4. You hear that your new neighbor has two children. You see one child, a girl, playing in the backyard. What is the probability that both children are girls? Using concrete materials, design an experiment that you could use to simulate this situation.

Standard 5. Problem Solving and Mathematical Reasoning

Students use mathematical reasoning and solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Problem Formulation (Students participate in the formulation of problems when given the basic statement of a problem situation.)

For example, students in high school who meet the standard will:

- Fill out the formulation of a definite problem to be solved.
- Extract pertinent information from the situation as a basis for working on the problem.
- Ask and answer a series of appropriate questions to find a solution and do so with minimum "scaffolding" in the form of detailed guiding questions.
- Identify mathematical problems whose solution leads to a useful resolution of given situations.
- State questions with answers that could be derived from the information given in a situation.
- Identify mathematical questions that arise from situations or models of situations (e.g., features of the work environment, manipulatives, sketches, graphs, tables, diagrams, and so forth).

Problem Implementation (Students make the basic choices involved in planning and carrying out a solution.)

For example, students in high school who meet the standard will:

- Formulate reasonable mathematical hypotheses or conjectures based on a general description of a situation and identify any missing or extraneous information that might help or hinder the resolution of the situation.
- Select a variety of strategies or approaches to solve problems.
- Make connections among mathematical concepts or skills in the solution of problems.

Problem Conclusion (Students provide closure to the solution process through summary statements and general conclusions and make connections to, extensions to, and/or generalizations about related problem situations.)

For example, students in high school who meet the standard will:

- Verify the accuracy and validity of the mathematical procedures used to solve problems.
- Express problem-solving strategies as a general rule and extend strategies to other situations.
- Formulate generalizations of the results obtained.
- Carry out extensions of the given problem to related problems.

Mathematical Reasoning (The students not only make observations and state results but also justify or prove why the results hold in general.)

For example, students in high school who meet the standard will:

- Show forms of mathematical reasoning and proof appropriate to the solution of the problem at hand, including deductive and inductive reasoning, making and testing conjectures, and using counterexamples and indirect proof.
- Differentiate clearly between giving examples that support a conjecture and giving a proof of the conjecture.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Make a study of student absences caused by illness and find predictors for students in the top 10 percent and the lowest 10 percent of frequency of absences. Look for predictors related to diet, amount of sleep, ratio of weight to height, and so forth.
2. Find the dimensions of three right triangles such that in each triangle the lengths of the three sides and the length of the altitude to the hypotenuse are all integers.
3. Suppose you work for an unstable company and your salary undergoes changes over a three-month period. In each of the following situations, discuss how your salary at the end of the three-month period compares with your salary at the beginning of that period.
   a. Your salary is cut 10 percent and three months later is raised 10 percent.
   b. Your salary is cut 20 percent and three months later is raised 25 percent.
   c. Your salary is raised 10 percent and three months later is cut 10 percent.

4. Describe at least two situations in which the result of a calculation involving measurement must be rounded up (e.g., 8.4 inches rounded up to 9 inches) and at least two situations in which the result of a calculation must be rounded down (e.g., $8\,^3/_4$ inches rounded down to 8 inches).
5. Develop a convincing argument to explain why the product of two negative numbers is a positive number.
6. Design a running track with two semicircular ends and two parallel straightaways. If the track is to be exactly 440 yards around, explain how you would decide the length of the straightaway sections and the diameter of the semicircles.
7. Explain how you could decide whether the following game is fair or unfair.

   This game is played with two ordinary dice of different colors, (e.g., one white and one red). Both dice are thrown at the same time. The player wins if the number on the white die is greater than the number on the red die. The player loses in all other cases.

8. A politician stated in a speech that an increase in a budget item from $10,000,000 to $40,000,000 is a 300 percent increase; therefore, a decrease from $40,000,000 to $10,000,000 is a decrease of 300 percent. Explain whether or not this reasoning is correct.
9. When a newspaper ad was redesigned, the original column length was increased by one-half. To keep the area of the new ad the same as the area of the original ad, the original width was reduced by two-thirds. Explain whether or not this is correct and justify your reasoning.
10. In an elementary school district, it was decided to spend at least 8,800 minutes of instructional time per school year on mathematics in each class. What questions would have to be asked and answered in order to develop an instructional plan to meet that requirement?

Standard 6. **Mathematical Communication**

Students communicate their knowledge of basic skills, understanding of concepts, and ability to solve problems and understand mathematical communications of others.

For example, students in high school who meet the standard will:

- Use basic mathematical vocabulary and terminology, standard notation and symbols, common conventions for graphing, and general features of effective mathematical communication styles.
- Use mathematical representations with appropriate accuracy, including numerical tables, formulas, functions, algebraic equations, charts, graphs, and diagrams.
- Interpret and explain the problem.
- Present mathematical procedures and results clearly, systematically, succinctly, and correctly.
- Describe and discuss mathematical ideas effectively both orally and in writing.
- Explain mathematical concepts or ideas clearly to peers or others.
- Read with understanding mathematical text and other writing about mathematics.

Assignments or tasks that might be used to collect evidence of student work toward meeting the standards:

1. Write a computer program to solve equations of the form $Ax + By = C$, in which A, B, and C are any three real numbers. Explain what the relationship between A, B, and C must be for the solution to be an integer.
2. Explain, using words and graphs, what effect adding, subtracting, or multiplying x^2 by a number has on the graph of $y = x^2$.
3. Write an explanation for a sixth grade mathematics class showing why you get the results that occur in the following investigation:

    Pick your favorite number from 1 through 9. Multiply it by 9. Multiply that answer by 12,345,679. How is your result related to your favorite number?

4. Make and defend a conjecture about the evenness or oddness of the product of two or more consecutive integers.

High School

This section contains examples of several mathematics tasks with samples of student work for each task. The tasks were done by students enrolled in algebra I, in geometry, or in a two-year integrated mathematics class. Each task consists of the following:

- *Task* gives the directions for completing the assignment.
- *Mathematics in the Task* identifies the Challenge Standards addressed in the task (usually more than one) and describes the specific mathematics that students will encounter in completing the task.
- *Setting* describes the situation (course work, in-class work, homework, and so forth) and the time frame in which the sample work was completed.
- *Samples of Student Work* describes the mathematics evident in each student's work.

Taken together, this collection of tasks is *not* intended to be a test or an assessment of the high school mathematics standards. The tasks do not cover the full range of mathematics that students need to know, understand, and be able to do. They are merely examples of what might be included in an assessment or given as classroom assignments.

Most of the tasks are quite small; generally, they are assignments that may be done in class, as homework, or over a one-week period. Because most of the work is from only a few classrooms and was not intended as a formal assessment, *this document does not identify performance levels for the tasks*. Teachers from different classrooms, schools, and districts need to work together to establish performance levels for formal assessments that will be used to determine whether a student has met the mathematics standards. No single task can provide sufficient information to assess whether a particular student has met a standard. A collection of work from a student will be needed to make a valid judgment.

The samples of student work included with each task are intended to be illustrative of high-quality work that can be done by high school students and may be considered as some evidence that the students are progressing toward meeting the mathematics standards. The work samples are not in each student's handwriting—they were typed into a computer. However, they include the actual words (and spelling) and calculations used by the student; and they reflect, as much as possible, the student's drawings, format, and placement of work. Comments about the work describe the mathematical concepts and the approaches the students used as well as any errors they made. Most errors identified in the students' work are minor. The commentaries do not include the way in which the classroom teacher addressed the errors with students. Most tasks include work from different students.

Teachers are not expected to write commentaries about an individual student's work similar to those in this section. Teachers and students, however, are expected to *think* carefully about the mathematical concepts for assignments and tasks they do in their classrooms and about the evidence of mathematical knowledge and understanding clearly demonstrated in each student's work. Teachers may want to use the sample tasks and student work shown in this section as they discuss what the standards mean, how they are reflected in the classroom program, and the way in which they are exemplified in the work students do.

Task: Product of Four Consecutive Integers

Determine whether the product of four consecutive positive integers can be a perfect square. Justify your conclusion.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

Students need to understand the terms *product*, *consecutive positive integers*, and *perfect square* and represent the relationships expressed in the problem by using numbers or symbolic equations or both. They need to use inductive or deductive reasoning or both to find the correct solution, explain their reasoning, and defend their conclusion.

Setting

The task was a homework assignment in an algebra I class to be completed over a one- or two-day period.

Samples of Student Work: Student A and Student B

Student A's work opens with the correct conclusion, and the work that follows defends that position with an implied but well-constructed indirect proof. The work indicates that the student understands the concepts of product, consecutive integers, and perfect square and recognizes the conditions (three cases) that must be met if the product of the four integers is to be a perfect square.

The work shows an examination of all three cases with correct use of algebraic representations for the four consecutive integers and performance of the basic algebraic operations of binomial multiplication and equation solving. When each case culminates in a mathematical contradiction, the student identifies those contradictions correctly: the first (shown in solutions one and two) contradicts the opening statement that the numbers are all defined to be integers; the second (shown in solution three) reveals that the statement is inherently false. The logical argument is finalized by the closing reaffirmation that the product of four consecutive integers cannot be a perfect square.

Student A's Work (Product of Four Consecutive Integers)

The product of four consecutive positive integers can't be a perfect square.

Let $A \cdot B \cdot C \cdot D = y^2$

Then one of the following must be true:

1. $AB = CD$; 2. $AC = BD$; or 3 $AD = BC$

The product of two must equal the product of the other two in order for the product of the four to be a perfect square.

Let $A = x$; $B = x + 1$; $C = x + 2$; $D = x + 3$

1. $AB = CD$

$x(x + 1) = (x + 2)(x + 3)$

$x^2 + x = x^2 + 5x + 6$

$\underline{-x^2} \quad \underline{-x^2}$

$x = 5x + 6$

$\underline{-x} \quad \underline{-x}$

$0 = 4x + 6$

$\underline{-6} \quad \underline{-6}$

$-6 = 4x$

$-6/4 = 4x/4$

$x = -\frac{3}{2}$ <— NO, does not work —> because both are not positive and not integers

2 $AC = BD$

$x(x + 2) = (x + 1)(x + 3)$

$x^2 + 2x = x^2 + 4x + 3$

$\underline{-x^2} \quad \underline{-x^2}$

$2x = 4x + 3$

$\underline{-2x} \quad \underline{-2x}$

$0 = 2x + 3$

$\underline{-3} \quad \underline{-3}$

$-3 = 2x$

$-3/2 = 2x/2$

$x = = \frac{3}{2}$ NO

3 $AD = BC$

$x(x + 3) = (x + 1)(x + 2)$

$x^2 + 3x = x^2 + 3x + 2$

$\underline{-x^2} \quad \underline{-x^2}$

$3x = 3x + 2$

$\underline{-3x} \quad \underline{-3x}$

$0 = 2$ No, this is a false statement because $0 \neq 2$

Because none of these three possibilities work, the product of four consecutive positive integers cannot be a perfect square.

Student B's work shows a clear understanding of the nature of positive consecutive integers and the conditions that must be met if the product of four such numbers is to result in a perfect square. The work also displays number sense and mathematical efficiency by correctly recognizing the combination of pair-wise products most likely to produce identical results; namely, the first integer times the fourth and the second integer times the third.

The work shows that the algebraic notation x^2 is assigned to the perfect square and variable names (A, B, C, D) to the integers. By failing to assign notation to the integers that defines their relationship to each other (i.e., n, $n + 1$, $n + 2$, $n + 3$), the student misses the opportunity of generalizing the results.

The work shows three examples using sequences of four positive consecutive integers (single-, double-, and triple-digit integers) in which integers are paired and products compared using the approach described earlier in the work. An inductive conclusion is stated at the end of the work. Although the correct conclusion follows from well-chosen and clearly constructed examples, the work could be enhanced by a generalization demonstrating that the result holds for all sequences of four positive consecutive integers.

Student B's Work (Product of Four Consecutive Integers)

If the product of four consecutive positive integers were to equal a perfect square (x^2), it would mean that if you multiply a specific two together you would get X and if you multiplied the other two numbers together you would also have X For example, if the numbers A, B, C, D were consecutive one of the combination would have to be true

$A \times B = X$ & $C \times D = X$

$A \times C = X$ & $B \times D = X$

$A \times D = X$ & $B \times C = X$

$(X)(X) = X^2$

It would most likely be $A \times D = X$ and $B \times C$ because A would be the least and D the greatest number. For this to happen is not possible whenever you multiple $A \times D$ and $C \times D$ you never get the same answer. One product is always 2 more than the other.

Numbers:	1, 2, 3, 4	$1 \times 4 = 4$	$2 \times 3 = 6$
	20, 21, 22, 23	$20 \times 33 = 460$	$1 \times 22 = 462$
	101, 102, 103, 104	$101 \times 104 = 10504$	$102 \times 103 = 10506$

As you can see you never get two products of the same value. So it is not possible for 4 consecutive positive integers to equal a perfect square.

Task: Analyzing Election Results

Show that there must be at least one misprint in a newspaper report on an election which read:

Yes votes	13,657	(42%)
No votes	186,491	(58%)

Include your work and organize it so that your conclusions are clear.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

In this task students need an understanding of the concept of percent and how percents are calculated from given data. Students will also need to form and express at least one "if-then" type of relationship designating certain data to be assumed true and, thereby, rendering other data false. Students need to state clearly their speculations about possible errors and indicate the corrections they make.

Setting

The task was completed in class with 30 minutes allowed for both the initial work and the revisions.

Samples of Student Work: Student C and Student D

Student C's work shows the use of estimation skills to compare the difference in the number of no votes and yes votes (a comparatively large number) with the difference in the reported percentages (a comparatively small number). Throughout this work sample, all calculations are correct, and explanations are presented clearly and economically.

The work goes beyond the notion of miscalculation and addresses the notion of misprint by suggesting two creative alternative adjustments to the printed statistics that could account for the discrepancies and make the report correct.

Student C's Work (Analyzing Election Results)

Yes votes 13,657 (42%)
No votes 186,491 (58%)

According to this data printed by the newspaper, the difference in the number of votes is 172,834. However, the difference in the recorded percentages is only 16%.

Using the given data, the results should actually be:

186,491 ÷ 3,657 = 200,148

186,491 ÷ 200,148 = 93.2%

13,657 ÷ 200,148 = 6.8%

Therefore, using the given numbers, the percentage of yes votes should be approximately 6.8% instead of the recorded 42%. The percentage of NO votes should be 93.2% instead of the recorded 58%. Consequently, the newspaper must have made at <u>least one misprint</u>

Going one step further, I believe that the newspaper misprint occurred in the numbers 186,491 and 13,657. The number 186,491 can be changed to 18,491 or the number 13,657 can be changed to 130,657. The results would then be:

18,491 ÷ 13,657 = 32,148		130,657 ÷ 186,491 = 317,148
18,491 ÷ 32,148 = 58%	or	130,657 ÷ 317,148 = 42%
13,657 ÷ 32,145 = 42%		186,491 ÷ 317,148 = 58%

By changing the numbers the results work out properly.

Student D's work demonstrates an essential grasp of the mathematics relevant to this situation. The mathematical calculations, in an algebraic format, are carried out on the left, with observations based on those calculations written on the right. First, the percentages are assumed to be correct; subsequently, the vote counts are correctly shown to be erroneous. Then the vote counts are assumed to be correct, with proper mathematical calculations proving that the percentages are faulty.

A degree of weakness exists in this sample because there is little communication that either introduces or specifies the problem. Nor is there any summary of results or a feeling of closure brought to the investigation.

Student D's Work (Analyzing Election Results)

Misprints

$$\frac{x}{200148} = \frac{42}{100}$$
$$100x = 8406216$$
$$x = 84{,}062.16$$

1) 42% of 200,148 is not 13,657. it's 84,062.16

$$\begin{array}{r} 186{,}491 \\ \underline{13{,}657} \\ 200{,}148 \end{array}$$

$$\frac{x}{200148} = .58$$
$$x = 116{,}085.84$$

2) 58% of 200,148 is not 186,491, it's 116,085.84

$$\frac{13657}{200148} = \frac{x}{100}$$
$$1365700 = 200{,}148x$$
$$x = 6.8\%$$

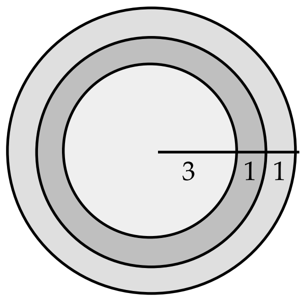
3) 13,657 is 6.8% of 200,148, not 42%.

$$\frac{186491}{200148} = \frac{x}{100}$$
$$x = 93.1$$

4) 186,491 is 93.1 % of 200,148 not 58%

Task: Target Probabilities

Juan believes that if you randomly threw a dart at the target shown below, the dart would more likely hit the bull's-eye (inner circle) than the outer ring. Do you agree with Juan? Assume that you hit the target every time and that the probability of where a dart hits is proportional to the area of each ring of the target. Explain your thoughts and prove him either right or wrong.



Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x	x	x	x	x

In this task students need to recognize the radius of a circle, apply the formula of the area of a circle using the appropriate radii, and identify areas that need to be compared. They also need to understand probability as a ratio of numbers and geometric probability, in particular, as a ratio of areas.

Setting

The task was completed in a geometry class period with about 20 minutes allowed for completion of all work and writing.

Samples of Work: Student E, Student F, and Student G

Student E's work shows a clear understanding of the underlying concepts involved in the task. The student carefully describes all calculations made in determining the circular areas and correctly relates the areas of the inner circle and outer ring to the relative probability that either will be hit.

An error is made, however, in that the student appears to construct the ratio of the areas as 1/1 and to regard this as the significant probability. A few sentences later, in summary, the student implies an understanding that there are two probabilities of the same value on which the correct conclusion is eventually based. In this piece the student explains why the picture tempts us to think that the inner circle is more likely to be hit, but stays with the stated generalization about the mathematical probability.

Student E's Work

Juan is wrong. The outer ring is the exact same area as the bull's eye. To figure this problem out, I used area. The area of the bull's eye is

$$A \approx \pi r^2$$
$$A \approx \pi 3^2$$
$$A \approx \pi 9$$
$$A \approx 28.27433388$$

Next I figured the area of the middle ring.

$$A \approx \pi r^2$$
$$A \approx \pi 4^2$$
$$A \approx \pi 16$$
$$A \approx 50.26548246$$

then I subtracted the bull's eye from the middle ring to give the area of the surface of that ring of the dart target

$$50.26548246 - 28.27433388 \approx 21.99114858$$

Then I figured the area of the outer ring

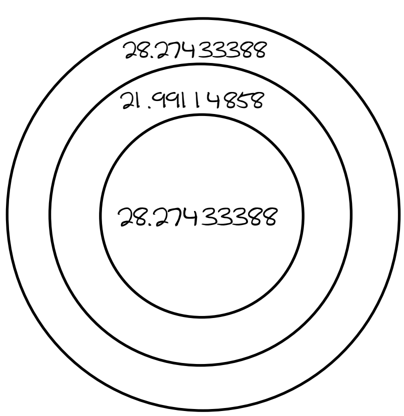
$$A \approx \pi r^2$$
$$A \approx \pi 5^2$$
$$A \approx \pi 25$$
$$A \approx 78.53981634$$

then I subtracted the 2nd ring (the 1st figure)

$$78.53981634 - 50.26548246 \approx 28.27433388$$

The area of the bull's eye and the outer ring are the exact same size. Therefore, the probability is 1 : 1.

But one thing to take into consideration is that the bull's eye is all in one place a greater whole section when the outer ring is more "spread" out because the bull's eye is in one place it might be easier to hit it but mathematically the probability is exactly the same. Therefore, Juan is wrong.



Student F's work shows that the student has a good intuitive sense of the situation. The significant circular areas are correctly calculated by substituting the appropriate radius into the proper formula. The correct subtraction is made to isolate the area of the outer circle. The student draws an appropriate conclusion by comparing the size of the areas involved.

Student F's Work (Target Probabilities)

Area of whole circle $\approx \pi r^2$
$\approx 3.14\ (3 + 1 + 1)^2$
$\approx 3.14\ (5 \cdot 5)$
$\approx 3.14\ (25) = 78.54$

Area of inner circle $\approx \pi r^2$
$\approx 3.14\ (3 \cdot 3)$
$\approx 3.14\ (9) = 28.27$

Area of outer circle $\approx 78.54 - (3.14(3 + 1)^2)$
$\approx 78.54 - (3.14(4 \cdot 4))$
$\approx 78.54 - (3.14(16))$
$\approx 78.54 - 50.27$
≈ 28.27

As you can plainly see, the areas of the two spots on the target that Juan is talking about are the same size. That means that you have the same chance of hitting the bullseye as you do of hitting the outer ring. Juan is wrong.

Student G's work shows the necessary and correct calculations of the circular areas. This part of the work would be substantially enhanced by adequate labeling or other communication regarding the purpose for these calculations.

The calculations of the probabilities for hitting the inner circle and outer ring are shown and adequately labeled. The final summary brings closure to the task and an answer to the question asked.

Student G's Work (Target Probabilities)

Big Circle:	$r \approx 5$
	$\pi r^2 \approx$ Area
Med circle:	$r \approx 4$
Area of all:	$\pi(5)^2 \approx 78.54$
	$\pi(4)^2 \approx 50.27$
Outer ring:	$\pi(5)^2 - \pi(4)^2 \approx 28.27$
Small circle:	$r \approx 3$
	$\pi(3)^2 \approx 28.27$
Med ring:	$\pi(4)^2 - \pi(3)^2 \approx 21.99$

$\frac{28.27}{78.54} \approx 36\%$ chance of hitting outer ring

$\frac{28.27}{78.54} \approx 36\%$ chance of hitting small circle

If a dart is thrown randomly at that dart board, there is an equal chance of hitting either the small circle or outer ring. Juan is wrong.

Task: A Parabola Group Investigation

You already know that $y = x^2$ is a parabola and how to shift it around. Investigate what happens when you change each of the constants, a, b, and c, in the equation $y = ax^2 + bx + c$.

Each group is responsible for a detailed, thorough report on this question. Your report should include at least the following:

- A description of how the group organized its investigation. What did you do? In what order? Tell exactly what each person did. Your method should show evidence of a systematic approach.
- Plenty of relevant examples, including T-tables, graphs, or diagrams where appropriate.
- A detailed and thorough summary of what you observed.
- Each individual's own evaluation of this project. Comment on how your group worked together, what you thought of the project, and how you think it could be improved. Comment also on your group's approach. What would you do differently? What would you do the same if you could start again?
- A brief presentation to the class (5 to 10 minutes) using overhead transparencies.

Mathematics in the Task

Standard 1 Number and Operations	Standard 2 Geometry and Measurement	Standard 3 Function and Algebra	Standard 4 Statistics and Probability	Standard 5 Problem Solving and Mathematical Reasoning	Standard 6 Mathematical Communication
x	x			x	x

This task requires students to investigate what happens when constants are changed in the standard form of a quadratic function. It requires that students have an understanding of and an ability to represent quadratic functions as equations, T-tables, and graphs. Students must communicate about the mathematics involved when they plan their work with others in small groups, when they record individually in writing their findings and conclusions, and when the groups present their conclusions to the class.

Setting

This task was done in small groups of four students near the end of the year in an algebra I class. The students worked on the task one day in class and completed it as homework over two nights. Group presentations were done during another class period. A group assigned each member a specific part of the task, and individuals shared their findings with the group. Each person was responsible for turning in a complete report containing the work done by the other group members and individually written summaries and conclusions.

Samples of Student Work: Student H

Student H's work includes the following parts: (1) the method of the investigation, which appears below; (2) analyses of changing each of the three constants and of changing all three (pp. 143–149); (3) an evaluation of the investigation (p. 150); and (4) copies of overhead transparencies the group used (pp. 151–152). Student H completed parts 1 and 3 independently and included work from her three colleagues plus her own work for Part 2. The method of investigation describes which task each member was assigned to analyze and the conditions the group agreed on to keep results comparable and why these conditions were made. Part 2 includes Naomi's analysis of changing *a*, Jan's analysis of changing *b*, Owen's analysis of changing *c*, and Student H's analysis of changing all three constants. Student H wrote her own summary statement on the bottom of Jan's analysis because Jan had omitted a conclusion. Student H's evaluation of the investigation includes some concrete suggestions for approaching the problem differently if it were to be done again.

Student H's Work

Group Investigation: Our method of investigation

In order to carefully analyze how each of the constants, a, b, and c changed the parabola individually, and then to determine how the parabola would be affected if all of the constants were changed, our group decided on the follow approach:

1. Each individual would graph the equation $y = x^2 + x + 1$. This equation which
   $a = 1$, $b = 1$, and $c = 1$, would be everyone's original graph and equation.
2. Since there were four of us in the group, three people would each take one constant to change and the fourth person would change all three of the constants at the same time. We then decided that:
   - Naomi would be responsible for analyzing the effects of changing "a"
   - Jan would be responsible for analyzing the effects of changing "b"
   - Owen would be responsible for analyzing the effects of changing "c"
   - I would be responsible for analyzing the effects of changing all the constants at once.

3. In order to keep Naomi's, Jan's, and Owen's results comparable, we agreed that they would only change their own constant and the other two constants remain equal to 1.
4. In order to keep all of our results comparable but still comprehensive, we decided to change our constants to the same values. Besides the original equation (where a, b, and c were all equal to 1) we would each graph another five equations.
   - to investigate when the constant(s) was/were changed to different positive values, each person would change the constant(s) to "3" and "5.
   - to investigate when the constant(s) was/were changed to different negative values, each person would change the constant(s) to "-3" and "-5
   - to investigate when the constant(s) did not exist at all in the equation, each person would change the constant(s) to "0"

Naomi

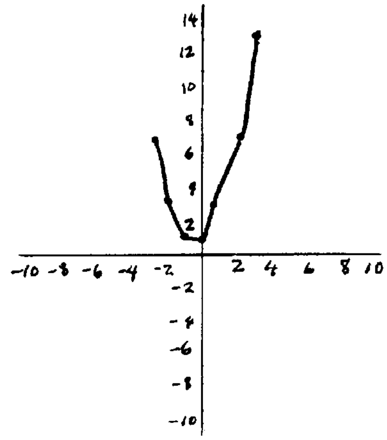
Changing "a"

$y = 1x^2 + 1x + 1$

a value = 3, 5, -3, -5, 0

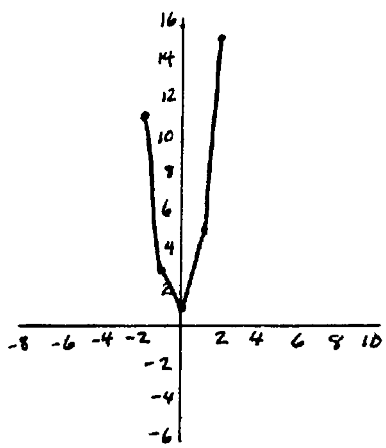
b & c = 1

x	y
3	13
2	7
1	3
0	1
-1	1
-2	3
-3	7
-0.5	0.75



$y = 3x^2 + 1x + 1$

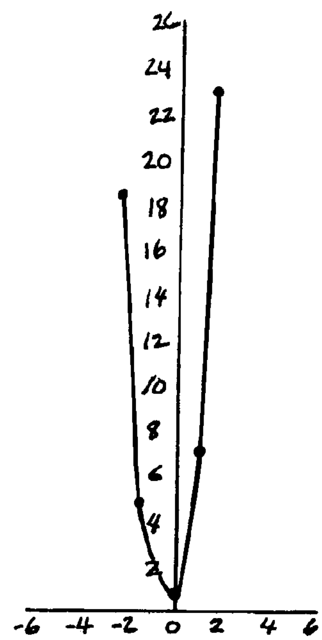
x	y
-2	11
-1	3
0	1
-1	5
-2	15



The parabola gets narrower when A gets further away from 0

$y = 5x^2 + 1x + 1$

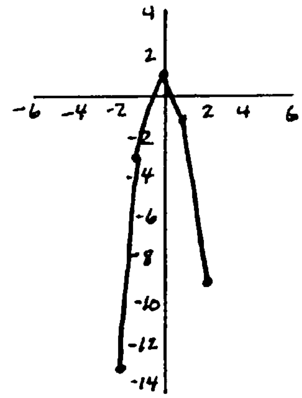
x	y
-2	19
-1	5
0	1
-1	7
-2	23



Naomi (page 2)

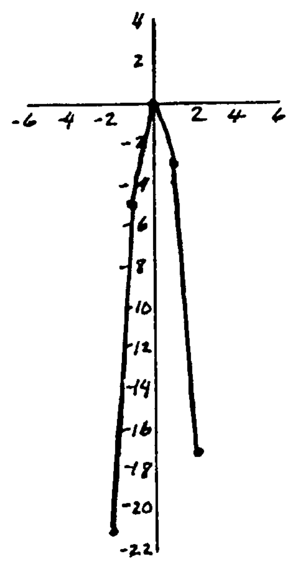
$y = -3x^2 + 1x + 1$

x	y
-2	-13
-1	-3
0	1
-1	-1
-2	-9



$y = -5x^2 + 1x + 1$

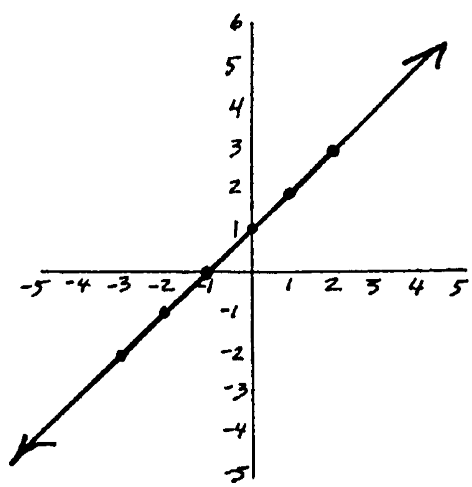
x	y
-2	-21
-1	-5
0	1
-1	-3
-2	-17



All the parabola has a vertex of (0,1)

$y = 0x^2 + 1x + 1$

x	y
-3	-2
-2	-1
-1	0
0	1
1	2
2	3



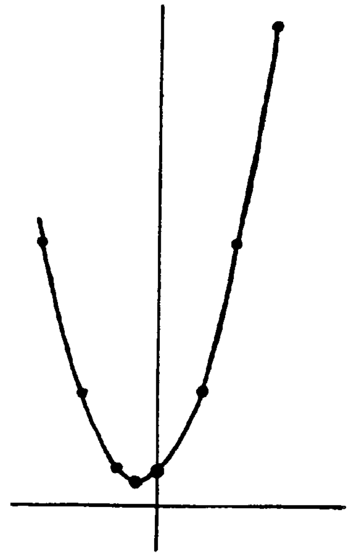
When "a" is zero, the graph is a linear line.

Changing b

Jon

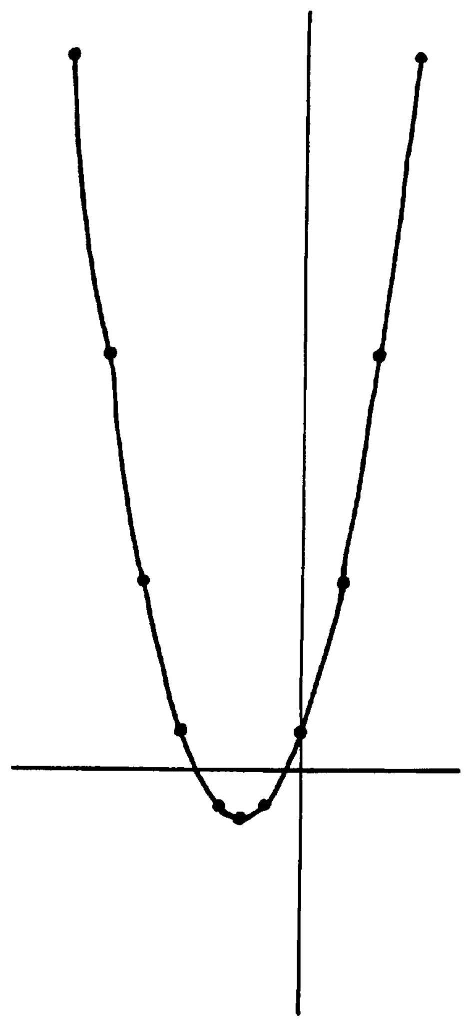
1. $b = 1, y = 1x^2 + 1x + 1$

x	y
3	13
2	7
1	3
0	1
-1	1
-2	3
-3	7
-0.5	0.75



2. $b = 3$
$y = x^2 + 3x + 1$

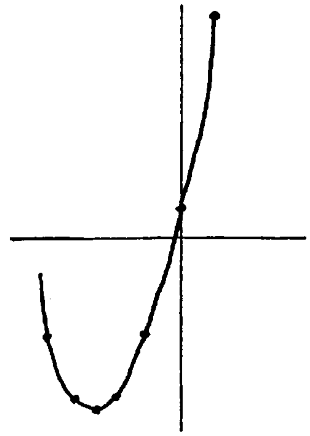
x	y
3	19
2	11
1	5
0	1
-1	-1
-2	-1
-3	1
-1.5	1.25



Jan (page 2)

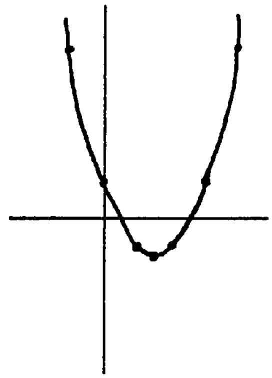
3. $b = 5, y = x^2 + 5x + 1$

x	y
1	7
0	1
-1	-3
-2	-5
-3	-5
-4	-3
-2.5	-5.25



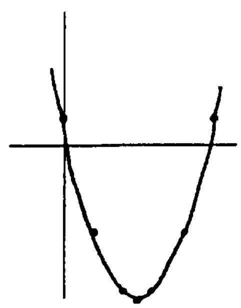
4. $b = -3, y = x^2 - 3x + 1$

x	y
4	5
3	1
2	-1
1	-1
0	1
-1	5
1.5	-1.25



5. $b = -5, y = x^2 - 5x + 1$

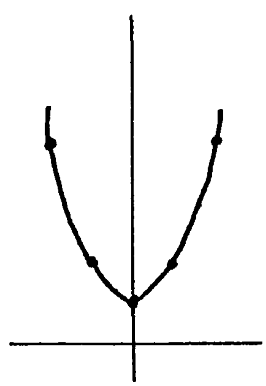
x	y
5	1
4	-3
3	-5
2	-5
1	-3
0	1
2.5	-5.25



Changing B
When you change the constant value of the variable b and leave the value of a & c as 1 . . . then the prababola itself doesn't really change just its position on the graph.

6. $b = 0, y = x^2 - 5x + 1$

x	y
2	5
1	2
0	1
-1	2
-2	5



The positive numbers produced negative vertices while the negative values for b produced positive ones. All the parabolas opened to the positive.

(written by Student H)

Owen

How the "c" variable affects the graph of the equation $ax^2 + bx + c = y$

$\{a, b, c\} = 1 \quad x^2 + x + 1 = y$

x	-3	-2	-1	-.5	0	1	2
y	7	3	1	.75	1	3	7

$\{a, b\} = 1 \quad c = \{-5\} \quad x^2 + x - 5 = y$

x	-3	-2	-1	0	1	2	-.5
y	1	-3	-5	-5	-3	1	-5.25

$c = \{-3\} \quad x^2 + x - 3 = y$

x	-3	-2	-1	-.5	0	1	2
y	3	-1	-3	-3.25	-3	-1	3

$c = \{0\} \quad x^2 + x = y$

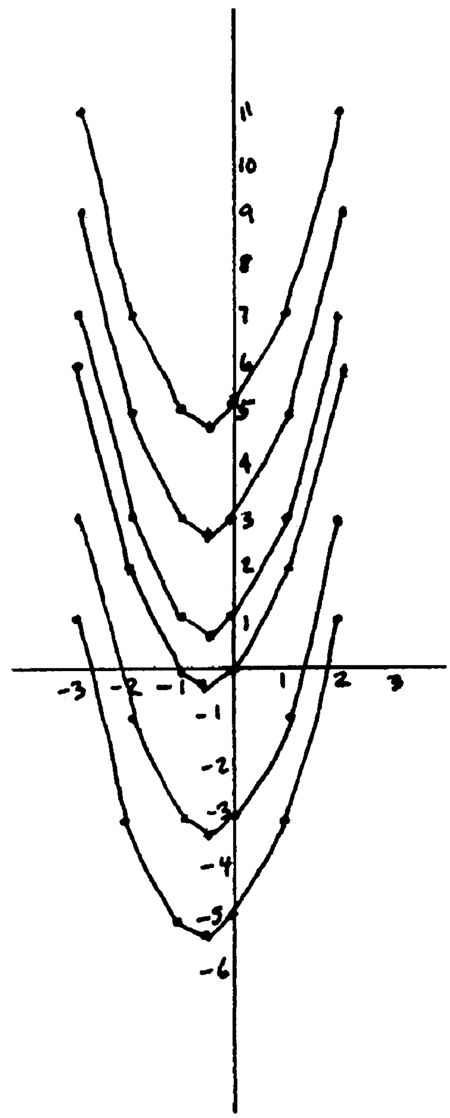
x	-3	-2	-1	-.5	0	1	2
y	6	2	0	-.25	0	2	6

$c = \{3\} \quad x^2 + x + 3 = y$

x	-3	-2	-1	-.5	0	1	2
y	9	5	3	2..75	3	5	9

$c = \{5\} \quad x^2 + x + 5 = y$

x	-3	-2	-1	-.5	0	1	2
y	11	7	5	4.75	5	7	11



This proves that the "c" variable is the y intercept of the equation.

Student H

Changing all variables to

(1) $a = 1, b = 1, c = 1$
(2) $a = 3, b = 3, c = 3$
(3) $a = 5, b = 5, c = 5$
(4) $a = -3, b = -3, c = -3$
(5) $a = -5, b = -5, c = -5$
(6) $a = 0, b = 0, c = 0$

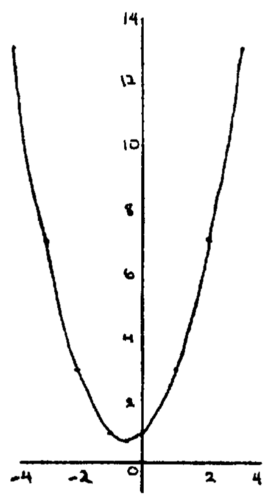
RESULTS

* the c value equals the y-intercept
* the vertex's x-value is always -0.5
* the y-values from when a, b, and c are positive and the y-values for when a, b, and c, are negative are opposites of each other.
* the y-value of the vertex is positive when a, b, and c are positive and the y-value of the vertex is negative when a, b, and c are negative.
* the further away from zero the values for a, b, and c are, the steeper the slope of the parabola.
* the vertex is "vertically" the value of a, b, or c divided by 4 units away from the y values or when $x = 1$ and $x = 0$.
* the value for a or b or both values controls the parabola's steepness.
* when a, b, and c, equal 0, the graph is the x-axis.

(1)

$y = x^2 + x + 1$

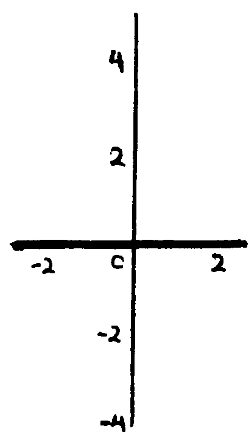
x	y
3	13
2	7
1	3
0	1
-1	1
-2	3
-3	7
-0.5	0.75



(2)

$y = 0x^2 + 0x + 0$

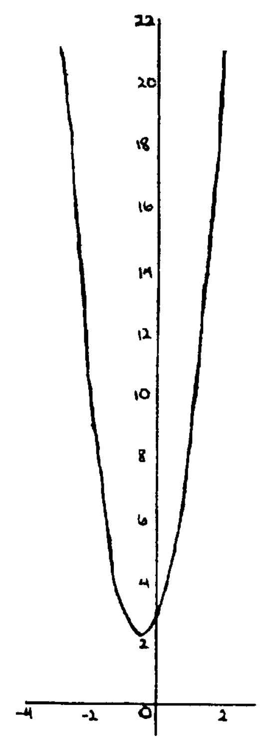
x	y
3	0
2	0
1	0
0	0
-0.5	0
-1	0
-2	0
-3	0



(3)

$y = 3x^2 + 3x + 3$

x	y
2	21
1	9
0	3
-0.5	2.25
-1	3
-2	9
-3	21



Student H
Changing all Variables
Page 2

(3) $y = 5x^2 + 5x + 5$

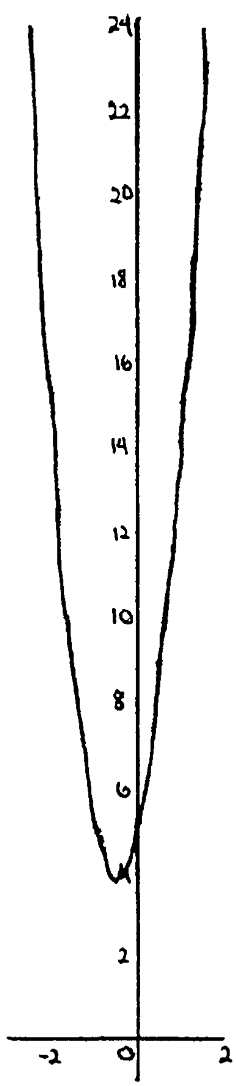
x	y
1.5	23.75
1	15
0.5	8.75
0	5
-0.5	3.75
-1	5
-1.5	8.75
-2	15
-2.5	23.75

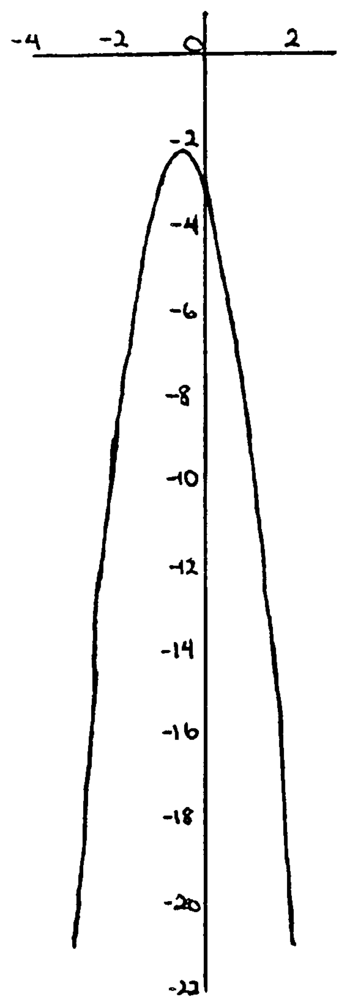
(4) $y = -3x^2 - 3x - 3$

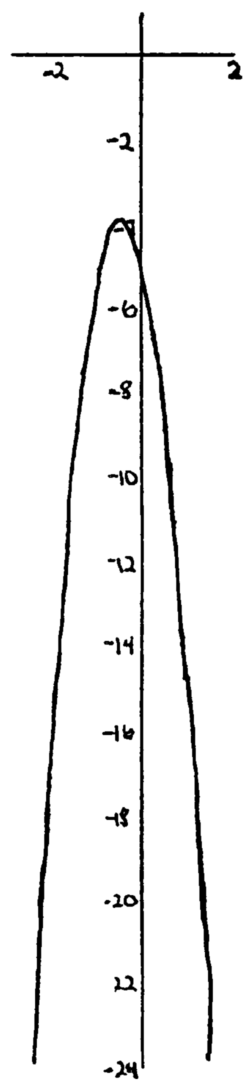
x	y
2	-21
1	-9
0	-3
-0.5	-2.25
-1	-3
-2	-9
-3	-21

(5) $y = -5x^2 - 5x - 5$

x	y
1.5	-23.75
1	-15
0.5	-8.75
0	-5
-0.5	-3.75
-1	-5
-1.5	-8.75
-2	-15
-2.5	-23.75







Student H

Parabola Group Investigation: EVALUATION

For this investigation, my group worked together and everyone had equal responsibilities, which each of us did well. The first day, each of us chose a specific constant or constants to change and we were in charge of graphing and analyzing the resulting graphs of the equation when only that one constant was changed and the other two kept constant, or, in my case, when all of the constants were changed at the same time. I think that this was a good and systematic approach, and because the numbers that we changed a, b, and c to were decided on by all of us to be -5, -3, 0, 3, and 5, our graphs were comparable to each others'. Also, since we also agreed to all graph the equation when all of the constants were 1, and that when the constant in the equation was not going to be changed to -5, -3, 0, 3, or 5, to keep that constant 1, the resulting graphs of each person were comparable to the other graphs of that person and any variations in the graph, or any common factors in the graphs, would be noticed.

I thought that this was a good project because it helped us see what each constant in the equation "$y = ax^2 + bx + c$" contributed to the graph and its location. We can use this knowledge later on when we graph parabolas to check and make sure that the parabolas have been graphed correctly by comparing each term of the parabola's equation to the part of the graph that it controls and seeing if these two things correspond.

The next time my group does this, I would like to keep the approach of having the constants changed to numbers that everyone in the group used, to have the unchanged variables remain a number that is always the same, and to have this "always-the-same" number replace all of the constants in a "control graph" and a "control equation" so that everyone has something to compare their other graphs to. However, the next time I would like to expand on our process and have a, b, and c changed to numbers which are not so close in value, such as to -12, -8, -4, 0, 8, and 12 and to have two control groups, where in one group, all of the constants are 1, so that the graphs in which constants were changed to positive numbers have a comparable graph and to have all of the constants in the second control group changed to -1 so that the graphs in which constants were changed to negative numbers have a comparable graph.

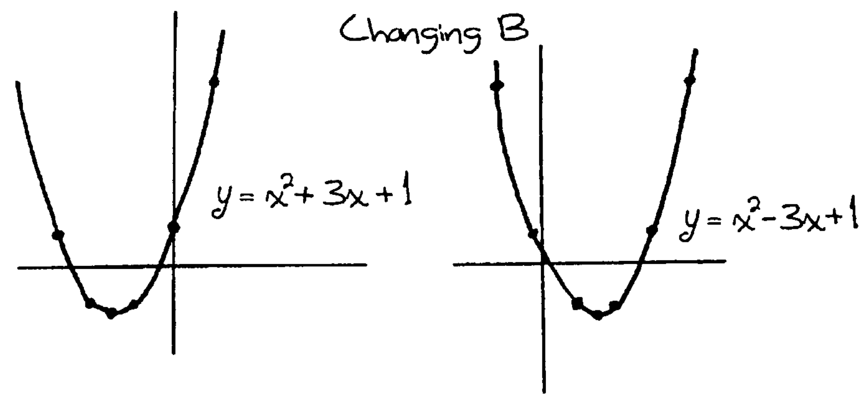
These were the overhead materials that the group used to present their findings to the class.

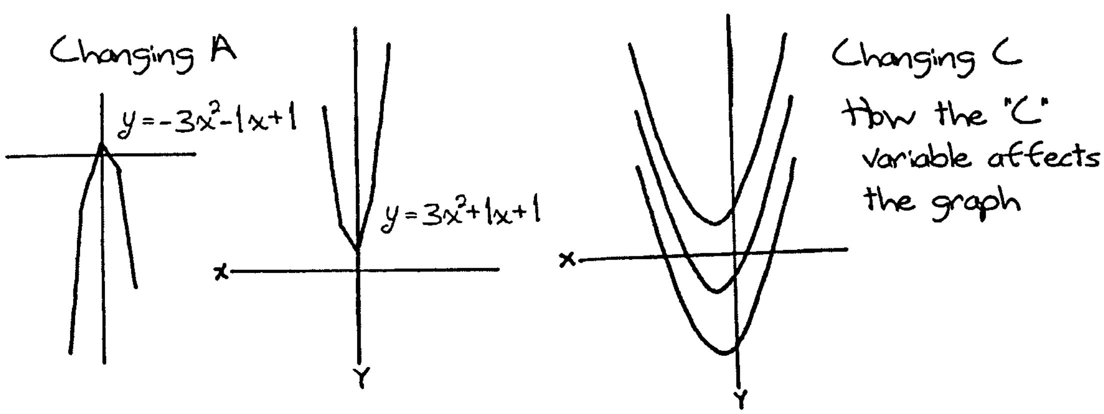
Results

Changing A –
It determines if the parabola opens up or down

Changing B –
It determines the location of the graph (right, left, up, down)

Changing C –
It determined the location of the y-intercept





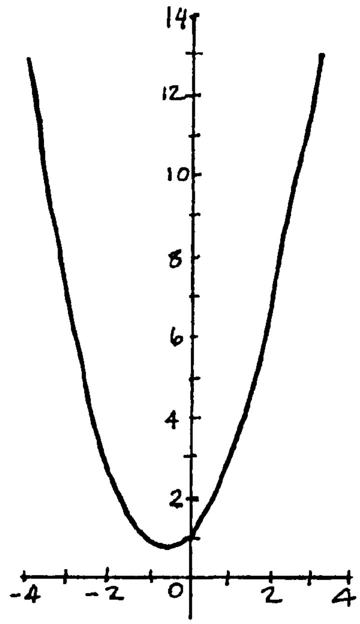
(Group overheads, p. 2)

Changing all variables

When a, b, and c have values of 1:

$y = x^2 + x + 1$

x	y
3	13
2	7
1	3
0	1
−0.5	0.75
−1	1
−2	3
−3	7

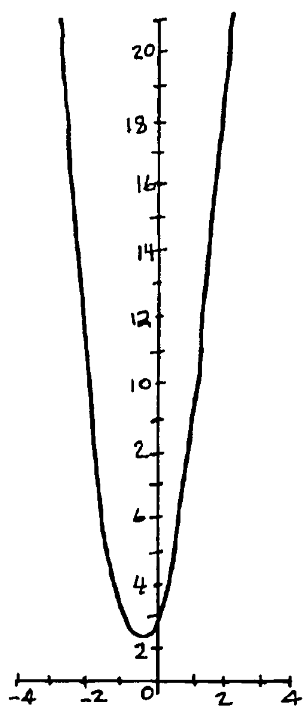


The combination of changing both the a and b variables determined the width and slope. The c variable still determined the y-intercept. The y-value from when a, b, and c are positive and the y-values from when a, b, and c are negative are opposite of each other.

When a, b, and c have values of 3:

$y = 3x^2 + 3x + 3$

x	y
2	21
1	9
0	3
−0.5	2.25
−1	3
−2	9
−3	21



When a, b, and c have values of −3:

$y = -3x^2 - 3x - 3$

2	−21
1	−9
0	−3
−0.5	−2.25
−1	−3
−2	−9
−3	−21